VOLUME III OCTOBER, 1938 NUMBER 4

# GEOPHYSICS

*A Journal of General and Applied Geophysics*

## VELOCITY DETERMINATIONS BY MEANS OF REFLECTION PROFILES*

C. H. GREEN†

ABSTRACT

While the idea of making an indirect determination of average sub-surface velocities by means of reflection profiles is far from new it is nevertheless considered worth reporting, inasmuch as it has been recently employed with fair success by the writer in areas beyond any wells that could be "shot" for direct velocity measurements.

Two examples of such surface velocity profiles are described—one located in the Means Field area of Andrews County, West Texas ,and the other about 8 miles south of the Refugio Field, in Refugio County on the Texas Gulf Coast.

### GENERAL

The simple mathematical background pertaining to the method is given with the help of Fig. No. 1. Thus for any reflection recorded with a spread of $X_1$ and time of $T_1$, we can write:

$$\left(\frac{VT_1}{2}\right)^2 = \left(\frac{X_1}{2}\right)^2 + Z^2. \qquad (1)$$

Changing the spread from $X_1$ to $X_2$ the same reflection will now have an arrival time of $T_2$.

The depth, $Z$, and therefore average velocity, $V$, are held constant, so that we can write as before that

$$\left(\frac{VT_2}{2}\right)^2 = \left(\frac{X_2}{2}\right)^2 + Z^2. \qquad (2)$$

Subtracting equation (2) from (1),

$$V^2 = \frac{X_1^2 - X_2^2}{T_1^2 - T_2^2} = A \text{ constant}. \qquad (3)$$

* Paper read at Annual Meeting, New Orleans, La., March 15, 1938.

† Geophysical Service, Inc., Dallas, Texas.

Thus in order to measure the average velocity down to any particular reflecting horizon it becomes only necessary to measure the reflection arrival times for any two different spreads. However, from (3) it is

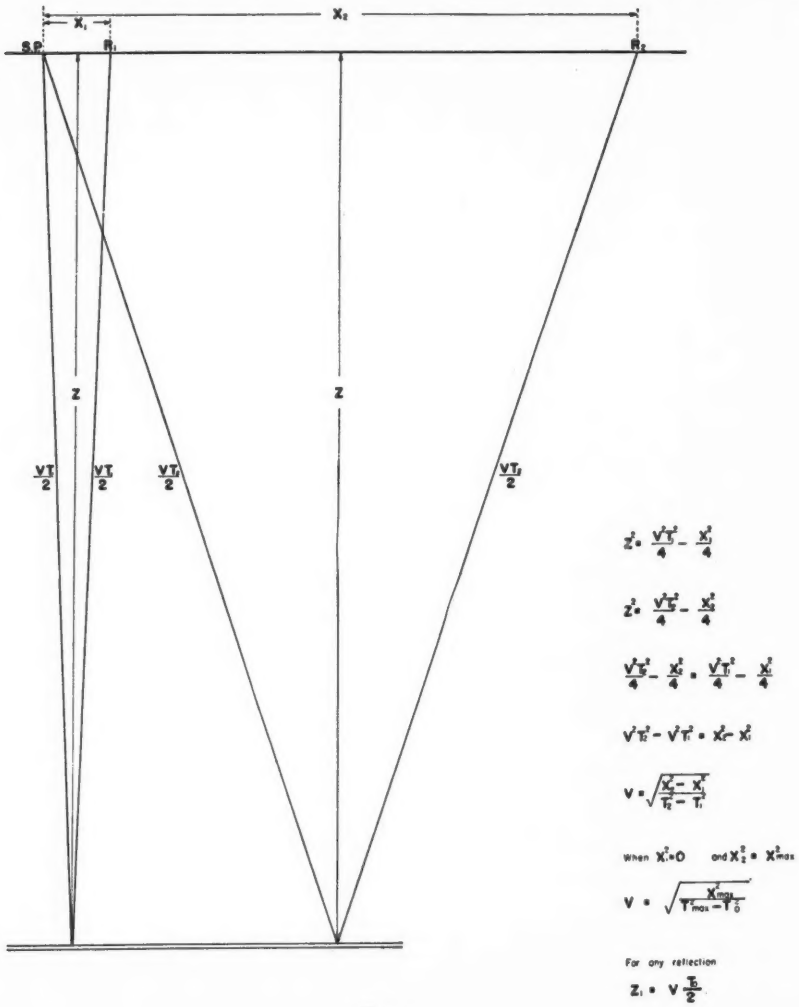



FIG. 1.

noted that there is a linear relationship between $X^2$ and $T^2$, so that instead of noting only two corresponding ratios of $X^2$ and $T^2$ in the field, it is recommended that several corresponding values be obtained, and to then draw the best straight line through all of the points

when plotted on linear cross section paper, so that the effect of a possible recording error for one particular point will then be minimized.

A glance at equation (3) reminds us that we are dealing with second order quantities as a result of taking the differences of large values, so that the determination of average vertical velocities vs depth by measuring the effect of a change of spread upon reflection arrival time has been frowned upon as an indirect, and therefore inaccurate method. However, even though indirect, it is felt that good reflection data can produce such a determination to within 3% accuracy. To accomplish this result it is necessary to comply with certain important set-up requirements in the field before deciding on the exact location for the velocity profile. These set-up specifications are enumerated as follows somewhat in the order of importance.

*a* At least one, and preferably several reflections must be known to be continuous over the coverage of the proposed profile.

*b* Weathering must be closely uniform for all of the recorder set-ups.

*c* Topographic changes must be a minimum over the several recorder positions.

*d* All depth shots should be in the same material and at the same approximate depth, so as to minimize the effect of variable shot hole conditions upon reflection reception times, as well as upon character and frequency.

*e* Reflecting horizons should be "flat," or at least, it is desirable to be able to shoot along strike.

It will be realized that for a series of recording positions to approach the above ideal conditions in surface as well as sub-surface characteristics, it becomes necessary to first of all gain an acquaintance with recording conditions over the prospect where the velocity information will later be applied. Naturally, such knowledge can only result from considerable recording experience over the area in the manner best suited to surface and sub-surface conditions which may be peculiar to the area. At this stage of progress it will also be necessary to assume a velocity-depth relation as a preliminary means of handling the reflection data. Upon entering a new area, the procedure sequence must be to first of all establish the most efficient reflection recording routine, and then after considerable scattered production has been acquired to then select the most favorable site for the velocity profile, having in mind the above mentioned specifications.

The importance of "flat" sub-surface can be minimized by arranging shot holes and recorder positions with common depth point location, and in addition to reverse shot holes and recorder positions about the same depth point. In other words, shot holes and corresponding recorder set-ups should be maintained equidistant from a center stake designated the depth point location.

In order to obtain a more or less uniform spacing of plotted $X^2$ vs $T^2$ points on the graph paper, it is suggested that the corresponding spreads, or $X$ values, be selected with this desire in mind.

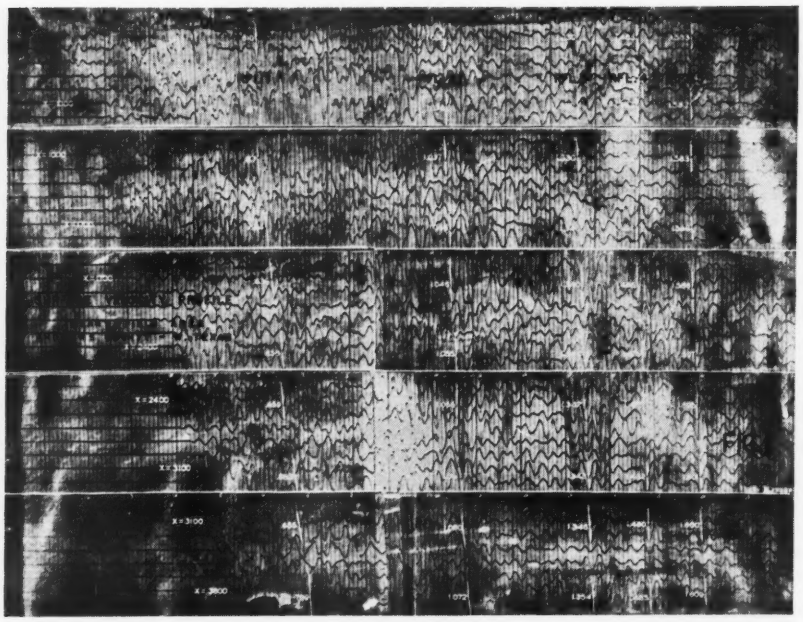


FIG. 2.

## MANIPULATION OF DATA

Surface profile data from two widely separated areas in Texas are shown—i.e., the Means Field area in Andrews County, West Texas, and the Refugio Field area, Refugio County on the Texas Gulf Coast.

Fig. No. 2 illustrates a profile of five Means Area records on which reception times for each of five reflections are noted on each spread as designated on the records. Similarly, Fig. No. 3 shows a profile of six seismograms recorded in the Refugio Area, on which

seven reflections continue across most of the spreads. Due to lack of time, both profiles were recorded in each case out of a single hole, but with recorder direction constant for all spreads.

The following tabulated data for the Means Area profile illustrates the method of preparing the reflection data for plotting, wherein $(T_r-t)$ is the recorded reflection arrival time as indicated on the several seismograms in Fig. No. 2, less the amount of weathering correction time.

VELOCITY DETERMINATION FROM VELOCITY PROFILE IN MEANS AREA, NORTH ANDREWS COUNTY, TEXAS

DATA SHEET

| *Mean spread* | 300 | 1000 1000 | 1700 1700 | 2400 2400 | 3100 3100 | 3800 |
|---|---|---|---|---|---|---|
| | | | *Reflection 1.* | | | |
| *Wx(t)* | .026 | .026 | .026 | .026 | .026 | .026 |
| *Tr(ind)* | .610 | .617 .617 | .634 .636 | .658 .659 | .683 .685 | .709 |
| Av. *Tr* | .610 | .617 | .635 | .658 | .684 | .709 |
| *Tr−t* | .584 | .591 | .609 | .632 | .658 | .683 |
| $(Tr-t)^2$ | .3411 | .3493 | .3709 | .3994 | .4330 | .4665 |
| | | | *Reflection 2.* | | | |
| *Tr(ind)* | 1.031 | 1.036 1.037 | 1.046 1.046 | 1.055 1.055 | 1.061 1.060 | 1.072 |
| Av. *Tr.* | 1.031 | 1.036 | 1.046 | 1.055 | 1.060 | 1.072 |
| *Tr−t* | 1.005 | 1.010 | 1.020 | 1.029 | 1.034 | 1.046 |
| $(Tr-t)^2$ | 1.0100 | 1.0201 | 1.0404 | 1.0588 | 1.0692 | 1.0941 |
| | | | *Reflection 3.* | | | |
| *Tr(ind)* | 1.330 | 1.330 1.330 | 1.340 1.340 | 1.344 1.342 | 1.348 1.346 | 1.354 |
| Av. *Tr* | 1.330 | 1.330 | 1.340 | 1.343 | 1.347 | 1.254 |
| *Tr−t* | 1.304 | 1.304 | 1.314 | 1.317 | 1.321 | 1.328 |
| $(Tr-t)^2$ | 1.7004 | 1.7004 | 1.7266 | 1.7345 | 1.7450 | 1.7636 |
| | | | *Reflection 4.* | | | |
| *Tr(ind)* | 1.463 | 1.463 1.461 | 1.464 1.465 | 1.475 1.475 | 1.482 1.480 | 1.485 |
| Av. *Tr* | 1.463 | 1.462 | 1.464 | 1.475 | 1.481 | 1.485 |
| *Tr−t* | 1.437 | 1.436 | 1.438 | 1.449 | 1.455 | 1.459 |
| $(Tr-t)^2$ | 2.0650 | 2.0621 | 2.0678 | 2.0996 | 2.1170 | 2.1287 |
| | | | *Reflection 5.* | | | |
| *Tr(ind)* | 1.583 | 1.583 1.583 | 1.588 1.588 | 1.597 1.595 | 1.601 1.600 | 1.609 |
| Av. *Tr* | 1.583 | 1.583 | 1.588 | 1.596 | 1.600 | 1.609 |
| *Tr−t* | 1.557 | 1.557 | 1.562 | 1.570 | 1.574 | 1.583 |
| $(Tr-t)^2$ | 2.4242 | 2.4242 | 2.4398 | 2.4649 | 2.4774 | 2.5058 |

Reference to Fig. No. 4 shows the various values of $(T_r-t)^2$ from the above table plotted against corresponding $X^2$ values, while Fig. No. 5 presents the same information from the Refugio profile. Note in both figures that the best straight line through the $(T_r-t)^2$ vs $X^2$ points for each reflection have been extended to intercept the $(T_r-t)^2$ axis in each case. By this expedient, we can predict the value of corrected $T^2$ at zero spread $(X=0)$ so that formula No. 3 can now be written for this special case of $X_2$ being equal to zero.

$$V^2 = \frac{X_{\max}{}^2}{T_{\max}{}^2 - T_0{}^2} \qquad (4)$$

in which $T_{max}^2$. is the time indicated by the best straight line at the maximum spread employed ($X_{max.}^2$) and $T_0^2$ is obtained from the intercept of the same straight line at the vertical axis as already mentioned.

The evaluation of Velocity, $V$, according to equation (4) is indicated on the straight line for each reflection in Figs. Nos. 4 and 5. Depth, $Z$, for each reflecting horizon is obtained simply from

$$Z = \frac{VT_0}{2}$$

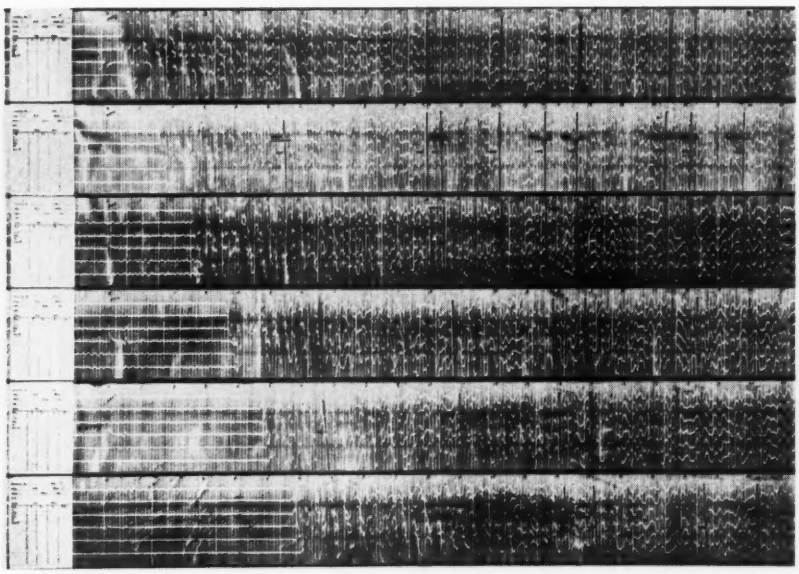


FIG. 3.

where $T_0$ is still the square root of the vertical axis intercepts associated with the straight lines in Figs. Nos. 4 and 5. The five depth values for the five reflecting horizons in the Means Area data are plotted against velocity in Fig. No. 6, which also shows the comparison with actual velocity data in the nearest well. This well, the Gill-Honolulu, Simpson No. 1, shot some time after the velocity profile, is located just east of Seminole in Gaines County, which amounts to about 18 miles separation from the profile location. It is to be noted that the average deviation from this well data amounts to about 5.5%, but as the velocity profile values are the higher, we can explain at least

a part of this difference on the basis of the velocity profile being in the direction of the Fort Stockton Uplift from the Simpson well, where still higher average velocities are encountered. In similar manner, velocity vs depth values for the Refugio profile are given in Fig. No. 7 and also compared with data from the nearest well in

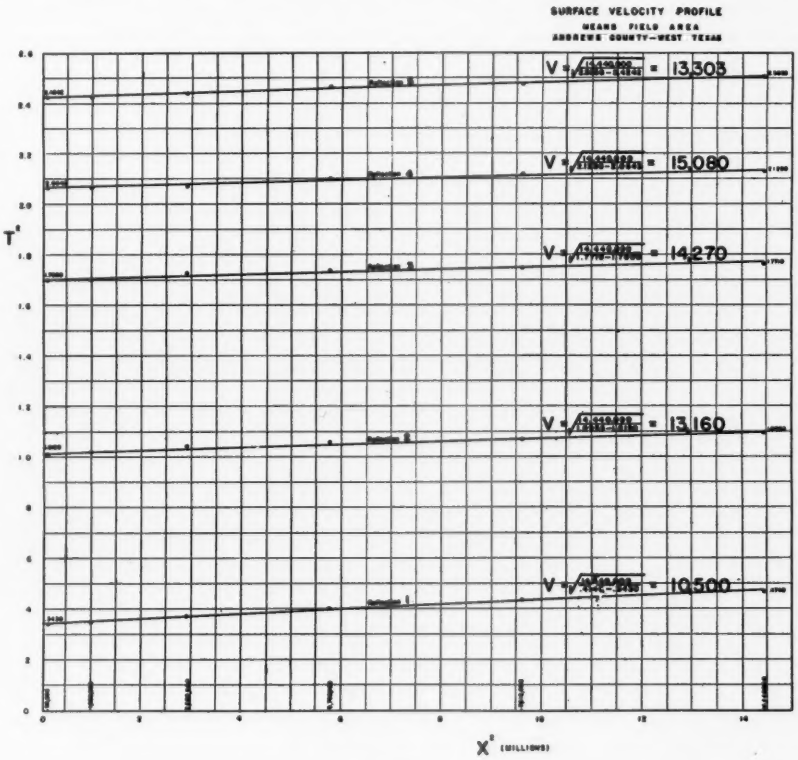



FIG. 4.

that vicinity, the Porter Drilling Company, Schmalsteig No. 1, which is located between Sinton and Neuces Bay in San Patricio County. In this case the separation amounts to about 15 miles, but being on the Gulf Coast where velocity changes are not apt to be abrupt and approximately on strike with each other, the comparison may be considered a fair one after all. The average deviation of the profile data from the well information amounts to about 2.5%. In both areas the vertical near-surface, or unweathered velocities as obtained from considerable recording experience with deep shot holes

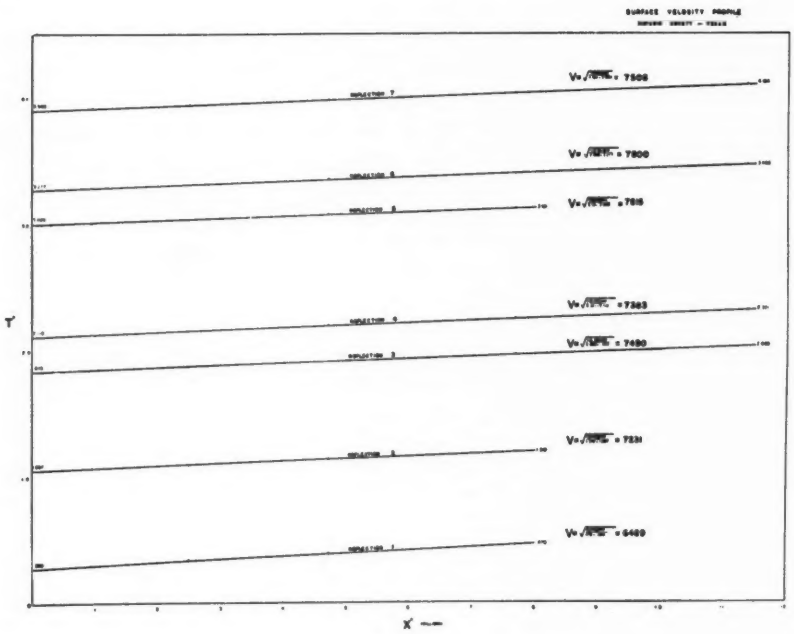

SURFACE VELOCITY PROFILE
T'
X'

FIG. 5.

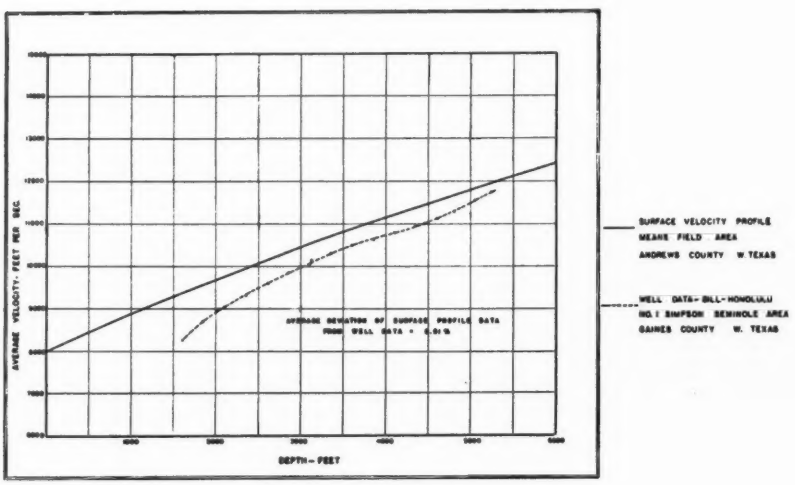

AVERAGE VELOCITY: FEET PER SEC.
DEPTH – FEET
SURFACE VELOCITY PROFILE
MEANS FIELD AREA
ANDREWS COUNTY W. TEXAS
WELL DATA – BILL-HONOLULU
NO. 1 SIMPSON SEMINOLE AREA
GAINES COUNTY W. TEXAS

FIG. 6.

were used for initial velocities ($Z=0$). It is often convenient to derive an equation from Velocity vs Depth curves such as have been obtained from the two velocity profiles—the customary equation forms being straight line, parabolic, or power series. In the case of the Means Area profile data for example, the parabolic form seems applicable—that is, of the form,

$$V^2 = CZ + K \tag{5}$$

for in Fig. No. 8 we have a plot of $V^2$ vs $Z$, which in being a straight line reveals the parabolic relationship. The constant "$K$" in equation

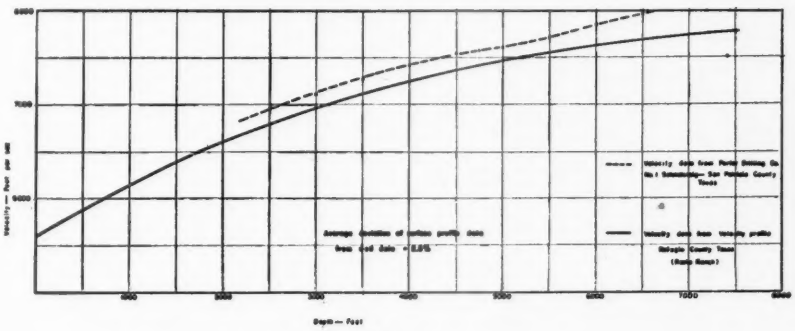


FIG. 7.

(5) as well as constant "$C$" are simply $V^2$ axis intercept and slope values respectively of the straight line in Fig. No. 8. Deriving values for the two constants in this manner, we can substitute same in equation No. 5 and rearrange to obtain the following,

$$V = 8{,}000\sqrt{1 + .000236Z}$$

where 8000 is recognized as the value of initial velocity.

## CONCLUSIONS

Both comparisons indicate that the surface profiles do not match the irregularities existing in the respective well data, nor is this to be expected without a great number of reflections to provide a corresponding large number of depth vs velocity determinations. However, for West Texas in particular, where present available velocity information of any kind is sparse, it would be difficult to predict just how far afield such irregularities would persist in unvarying amount as one progressed away from a reference well, such as we have in the

Gill-Honolulu well. Moreover, the proper incorporation of these irregularities in depth calculations for work either up or down dip from the well would be interfered with by the unknown effect of varying overburden. Thus, if we discount the importance of these minor variations for reconnaissance work at least, we can then say that the $V$ vs $Z$ lines for both profiles are closely parallel, or possess similar gradients to the well data in each case. As already pointed out,

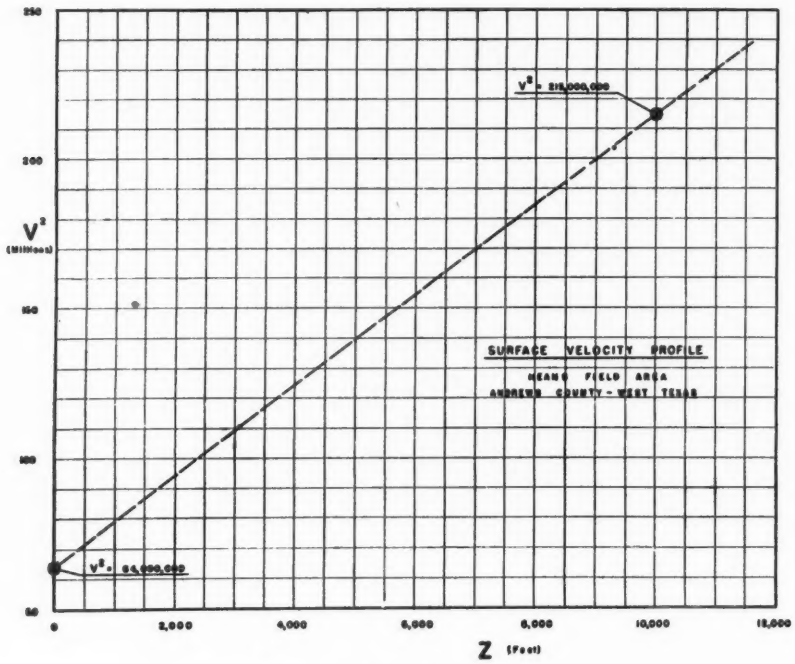



FIG. 8.

we have about 5.5% average variation between Means Area profile and corresponding well data, which could be at least partially due to the fact that the profile is situated about 18 miles south of the well, but even accepting this much difference as an actual error, the close parallelism of gradients in both forms of data will result in not more than 2.5% closure error. By averaging the results from several profiles in an area, it is expected that still greater accuracy could be obtained.

It is probably true, especially for West Texas, that the profile method is subject to some error due to much more definite stratifica-

tion than on the Gulf Coast, where intervening beds are relatively less consolidated. Because of this condition it follows that there is probably a more pronounced difference between horizontal and vertical velocities through any particular portion of section. Thus in West Texas the reflection paths to and from relatively shallow horizons would be subject to greater curvature or bowing away from straight line travel, which would naturally result in some distortion of the $X^2$ vs $T^2$ relation for very early reflections. It is again admitted that direct velocity measurements resulting from shooting of wells, when properly executed, are preferable to the indirect results from Velocity Profiles. However, in the absence of the former type of information, the latter is recommended as a fair substitute, and might even become very useful for short distances from a reference well where stratigraphic changes are occurring.

ACKNOWLEDGMENT

It is desired to acknowledge the assistance of Messrs. T. N. Walsh and Barney Fisher, both of Geophysical Service, in preparing the field data, as well as to Eugene McDermott for suggestions.

# ELECTRICAL PROSPECTING WITH NON-SINUSOIDAL ALTERNATING CURRENTS*

S. S. WEST†

ABSTRACT

A method of electrical prospecting using alternating current of rectangular or other non-sinusoidal waveform is described. The changes in waveform caused by anomalous subsurface structure are determined by a null method in which the detected electromotive force is balanced against an electromotive force produced by passing the output of a standard oscillator through an adjustable network. This process of measurement (applicable also to the transient method) makes possible a useful representation of the data in terms of two-dimensional contours.

The measurement of electrical resistivity in the earth has long been an important division of geophysical prospecting, but it has been necessary that the current source have an output of simple form, since the distribution of resistivities in the earth is usually complex enough without rendering interpretation still more difficult by complicating the apparatus. For this reason direct current and sinusoidal alternating current and, more recently[1,2] the step-form of transient have been used, because these possess the greatest mathematical simplicity. However, any non-sinusoidal alternating current whose waveform has a sufficiently simple representation in terms of Fourier series can be used and should have all of the theoretical advantages of the transient while being free of many of the practical disadvantages. Such an alternating current method bears some resemblance to the transient method and can make use of much of its experimental arrangement. The object of this paper is to describe some work carried out by the author in this field and to make clearer what is to be expected from this type of prospecting.

## APPARATUS

Non-sinusoidal alternating currents are most easily applied to prospecting by passing current between two electrodes imbedded in the earth and observing the electromotive force in a circuit containing the part of the earth between two other electrodes at a distance from

* Paper read at Annual Meeting, New Orleans, La., March 15, 1938.

† Subterrex, Houston, Texas.

[1] J. C. Karcher and Eugene McDermott; *Geophysics*, *5*, 64–77 (1934–35).

[2] L. Statham; *Geophysics*, *1*, 271–277 (1936).

the first pair. The best type of electrode system arranges four electrodes in a straight line, with the detecting electrodes outside the current electrodes, as in Fig. 1A. The rectilinear arrangement is desirable because it makes possible the use of a single line of electrode positions, perhaps laid out beside a road or on a path cut through brush, and because the wires can be dragged from one position to another without reeling them up. However, it should be observed

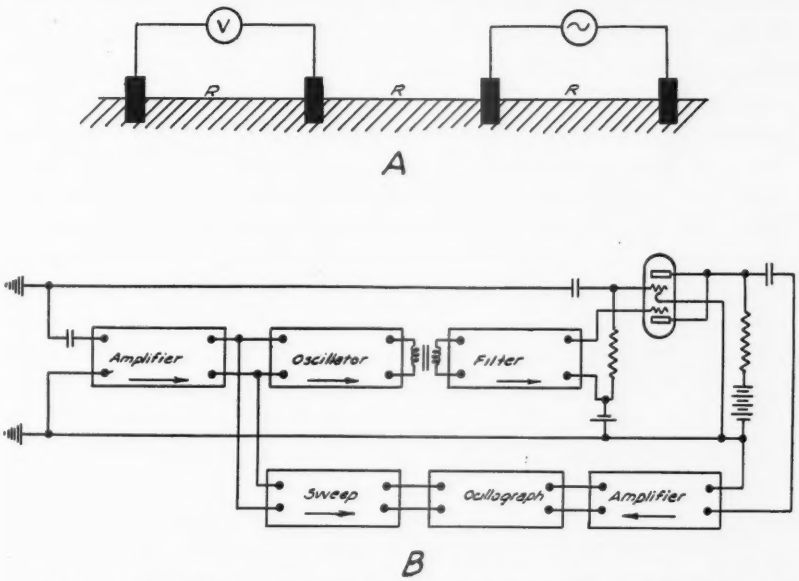



FIG. 1. *A* shows the electrode configuration, with the detecting electrodes at the left and the current electrodes at the right. Usually $R = 1000$ feet. *B* shows the detecting and analyzing circuit, which is represented in *A* as a simple voltmeter. Arrows indicate direction of control or propagation.

that interpretation is more complicated with the current spread parallel to the detecting spread than if these were perpendicular. This may be seen from the formulas derived by Foster.[3]

Placing the detecting electrodes outside the current spread is to be recommended for any method in which is used an alternating current or a transient. In these cases, the conventional Wenner electrode configuration would place the wires of the two circuits so close together that the electromotive force induced in the detecting circuit by the current flowing in the external current circuit would tend to

[3] R. M. Foster; Bell System Technical Journal, *10*, 408–419 (July, 1931).

be as large as that due to currents flowing in the earth. Nevertheless, an arrangement which places the detecting electrodes outside is not as sensitive as the Wenner arrangement to resistivity contrasts between horizontal strata, and it can also be easily seen that, for the pure DC case at least, with given electrode separation, there will always be a depth at which a horizontal interface gives no indication of its presence.

Our usual electrode configuration consisted of four electrodes in a straight line with equal spacings as in Fig. 1 A, the electrode interval being 1000 feet. The connecting wires were laid down along the straight line determined by the electrode positions. The two circuits were thus separated by 1000 feet of space, so that the direct induction had a sufficiently low value as long as there existed in the immediate neighborhood no grounded metallic conductors like fences or pipelines. The central space was sometimes made smaller or larger in order to obtain special conditions which would give the detected electromotive force a more easily measurable form.

In the transient method a square step of current is produced by closing a switch in series with a battery and the earth. For electrode separations of a thousand feet or so the self-inductance of the earth is so small that the current attains a maximum value nearly instantaneously with respect to the transient times to be measured. However, there are practical disadvantages in such a procedure. It is here necessary to amplify the received signal by means of an amplifier with low phase-distortion and frequency response constant to zero frequency, and some means such as porous pots must be used at the detecting electrodes to minimize the effects of electrode polarization. Moreover, one is limited to the time of a single cycle in the production of an image of the potential-time function on an oscilloscope screen, so that photographing this requires a bright image and fast lens.

In the associated alternating current method we applied to the current electrodes an alternating electromotive force of rectangular waveform and a frequency of 50 cycles, which is much the same thing as opening and closing the switch of the simple transient circuit with that frequency. The electromotive force observed at the detecting electrodes has 0.01 second to come approximately to a steady state in each half-cycle. Transients to be observed on the Gulf Coast with such an electrode configuration do not usually reach a steady state in this time, but the arbitrarily chosen frequency remains

quite satisfactory for our method of measurement, for we are here considering instead an AC steady state. Here the amplifier did not have to pass frequencies down to zero, so that it was possible to use standard resistance-capacity coupling of its stages. By using a sweep circuit of conventional design, one or more cycles of the modified wave could be kept stationary on the screen of an oscilloscope connected to the output of the amplifier. A condenser was inserted into the detecting circuit to eliminate the effect of contact potential differences at the electrodes.

The current source was a thyratron relaxation oscillator, which was controlled by means of a 50 cycle tuning fork. The plate battery consisted of 120 to 360 volts of ordinary lead storage cells. The average current into the ground was between 1 and 2 amperes.

Statham states in his paper[2] that differences in the shape of the transient between one location and another nearby are so small as to be less than the thickness of the line on the cathode-ray oscilloscope screen. However, Karcher and McDermott[1] record such variations, and our own observations also show that they exist. If the linear electrode arrangement with equal separations is moved by only a fraction of the thousand-foot electrode spacing, the change in the transient can often be easily seen on the oscilloscope screen or on a photograph of it. The modified rectangular waveform shows similar changes.

To measure such changes, not only when they are less than the thickness of the cathode-ray line, we set up a circuit[4] by which the signal from the detecting electrodes was balanced against a signal of nearly the same waveform and the difference amplified and observed on the cathode-ray tube. The balancing voltage was obtained by impressing upon a filter network a rectangular wave of the same frequency and as nearly as possible the same phase as the voltage across the input of the main amplifier. By varying the elements of the network, the waveform of the balancing signal could be so changed that the observed difference from the earth signal was small over a large part of each half-cycle. Of course no finite network can produce the same modification of waveform as does the semi-infinite solid conductor, but a sufficient approximation is possible. From a knowledge of the values of the filter elements, the approximate waveform can be reconstructed mathematically. It is then a matter of choice whether to plot the voltage-time curve and make measurements

[4] S. S. West, patent pending.

from it or to calculate the values of whatever characteristic functions may be desired. Another method is to set to fixed magnitudes all filter elements but one and take the magnitude of this as characteristic of the shape of the received signal, when the last element and the amplitudes have been adjusted for the best possible balance.

The functional circuit of the amplifier and analyzer is shown in Fig. 1 B. The dual triode of the first stage of the amplifier permits mixing the signals without affecting the synchronization of the oscillator. The use of common batteries and plate resitor and of nearly identical tubes in this stage assures that the two voltages which are being balanced have passed through nearly identical circuits up to the point of mixing. The oscillator is of the thyratron relaxation type and supplies a rectangular wave to the adjustable network. It is synchronized by applying the amplified earth signal to its grids through a transformer.

This arrangement can detect rapidly and easily changes in the shape of the modified rectangular wave, though it is not wholly suitable for absolute determinations of the waveform. Some forms of distortion are difficult to eliminate, but they are constant in nature and degree and were tolerated for the sake of simplicity of the apparatus. Greater refinements are not justified in the present state of the theory.

## THEORY

The general scheme in electrical exploration involves the earth as a mutual impedance between two sets of terminals, one of which is used to supply energy and the other to detect or measure energy. This view ignores the possible presence of electric sources within the earth, as may be due, for instance, to electromotive forces generated by polarization effects between media of different chemical constitution. It has never been demonstrated, however, that these effects are appreciable.

Where the energy input does not vary with the time, the earth acts as a resistive coupling only. In that case the electric intensity is the derivative of a scalar potential function:

$$E = - \nabla \phi.$$

In a varying state, as for instance, with a sinusoidal input, to the right side of the above expression must be added the negative time derivative of a vector potential function:

$$E = - \nabla \phi - \dot{A}.$$

The potential $\phi$ is due to the presence of electric charges, which exist in appreciable amounts only on the survace of electrodes and on the boundaries between different conducting media. It is the effect of the latter charges which indicates the presence of such boundaries.

Very different is the situation with regard to the vector potential, for every elemental current filament is a source of this potential and the effects are due to an integration throughout space instead of merely over the boundaries between different media, as is the case for the scalar potential. The earth, then, regarded as a reactance, is very complicated and cannot be represented by a network even for the homogeneous case.

From the general equation $E = -\nabla\phi - \dot{A}$ alone one can gain some insight into the nature of the transient observed with the four-electrode system. The measured electromotive force is the integral

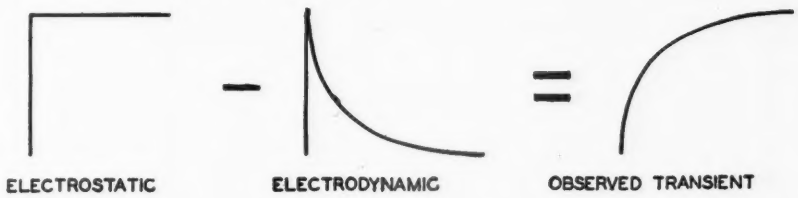



FIG. 2. The elements composing an earth transient. Ordinates represent electromotive force and abscissas time. The electrostatic part depends upon the applied potential difference only, but the electromagnetic part upon the rate of change of current in the ground.

of E over the part of the detecting circuit between the electrodes. The electrostatic portion of $E$, which depends on the scalar potential $\phi$, is established instantaneously, or at least with the velocity of light, which is practically the same thing for distances of a few thousand feet. The electrodynamic part of $E$, derived from the vector potential, is opposed to the electrostatic part and has its maximum at zero time, decreasing in absolute magnitude as the system approaches a steady state. The time variations of the corresponding parts of the measured electromotive force are shown in Fig. 2. The shape of the summation curve is not always smooth except for a homogeneous earth. Actually the electromotive force receives such a contribution from each current filament, the time variation being determined by the resistivity and geometry of the region in question. While the complete transient may have a shape like that of Fig. 2,

nevertheless there are cases where a hook or step or sudden change of slope may be present. Though the rectangular alternating current does not usually permit the electromotive force in the detecting circuit to reach the transient steady state, the effect upon each half-cycle is much the same as the effect upon the transient, with regard to change of shape.

## EXPERIMENTAL RESULTS

Some modifications of the rectangular waveform are shown in Fig. 3, where abscissas represent time and ordinates electromotive force. At (a) is a normal form, (b) is characteristic of the effect

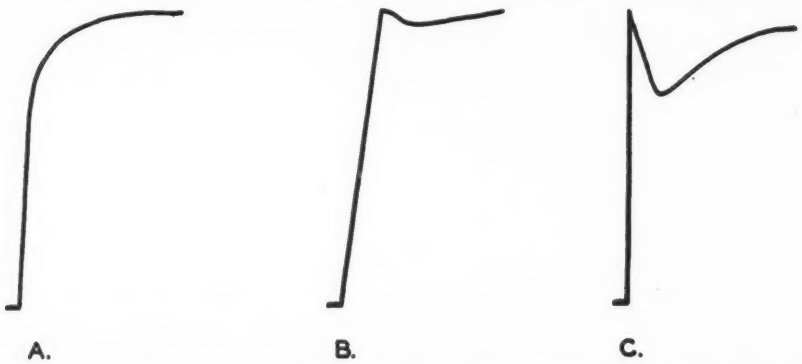



FIG. 3. Characteristic waveforms obtained with a rectangular input current, as copied from photographs. Only one-half cycle is shown in each case. Ordinates represent electromotive force and abscissas time. *A* is a normal form, *B* shows the effect of a pipeline, and *C* shows a type of hook which can occur where no artificial conductors are present. The waveforms approximate the transients to be obtained at the same locations.

produced by a pipe-line, and (c) shows how a hook can occur at a great distance from any buried metal. These last two types may be explained qualitatively as being due to a sum of several components, of the form of Fig. 2b, but with more rapid or less rapid change. In a great many cases, also, the initial increase of electromotive force contains a sudden change of slope, which is usually detectable only by the widening of the cathode ray line as the velocity of the spot becomes smaller. At present very little can be done in the way of quantitative explanation.

Inasmuch as there exists no mathematical analysis yielding a solution which can predict the shape of the transient or of the modified

rectangular wave for any useful cases, it is not very important what quantity is chosen as characteristic of the waveform. The only necessary conditions are that this quantity be determined uniquely by the structure of the subsurface and be sensitive to changes in it. Therefore we chose for each prospect fixed values for all but one of the parameters of the balancing network. By means of the remaining parameter, the observed electromotive force was then balanced out from

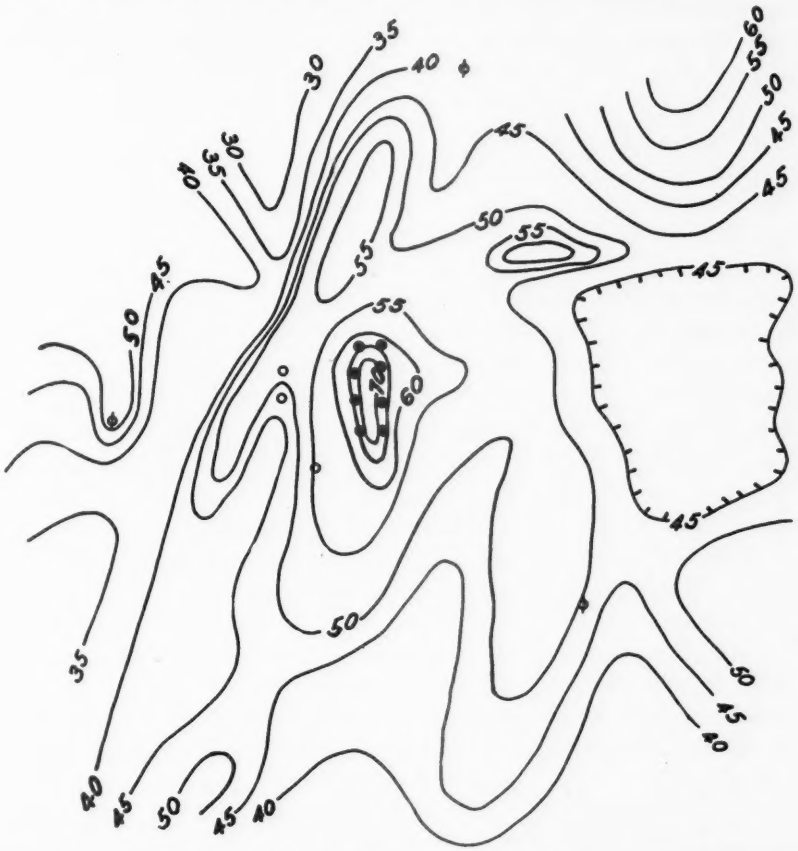



FIG. 4. Contour map obtained by varying one element of the filter network. Numbers on contour lines are dial readings. Wells are indicated by the conventional signs.

approximately 60° to nearly 180° after the beginning of the half cycle. The dial readings corresponding to the values of this parameter required for balancing the longest possible part of the cycle can then be plotted on a map and contours drawn. Such a map is shown in Fig. 4.

This map shows as an electrically anomalous closed structure the region in which are the producing wells, a phenomenon not often and particularly not in this case to be explained by the presence of cased wells and pipelines, since enough high readings to show the closed structure are obtained at considerable distances from buried metal. Contours of this form have been found at many other prospects, sometimes associated with known deposits of petroleum, as here.

One explanation of such anomalies has been offered by Rosaire[5] in a paper presented at this meeting. It is possible that they may indicate geological structures in which petroleum can collect, but whether this is so is still very uncertain. The idea that the later rise of the transient and hence the later part of the cycle in the case of rectangular alternating current depend entirely on the current flowing in deeper layers is certainly not correct. It would be better to say "more conductive" rather than "deeper" layers, and one must avoid thinking of a definite depth corresponding to the indication. For frequencies for which the electromotive force of each half cycle of received signal approaches the transient steady state, one can consider the half cycle as the sum of a square step and an approximate exponential increase, so that three quantities will characterize the shape. These are the total amplitude, the amplitude of the square step and the time constant of the exponential function. In general all three of these vary in the same way along a profile.

However, regardless of this lack of means for interpretation, the method that has been described permits for the first time rapid field measurements and a satisfactory representation of the data in two-dimensional contours in both the non-sinusoidal alternating current and transient methods. When the development of a method of interpretation justifies it, better balancing networks can be used, perhaps with a sum of several exponentials to approximate the whole of the wave.

[5] E. E. Rosaire, Shallow Stratigraphic Variations over Gulf Coast Structures, *Geophysics*, Vol. 3, No. 2, March 1938, pp. 96–115.

# THE RESOLUTION OF COMBINED EFFECTS, WITH APPLICATIONS TO GRAVITATIONAL AND MAGNETIC DATA*

THOMAS A. ELKINS† AND SIGMUND HAMMER†

ABSTRACT

A simple but rigorous and quite general mathematical method is given for finding the minimum separation of two nearby bodies at which their observed combined effect indicates the presence of two separate bodies. Geophysical applications of the method are illustrated by investigating the resolution of gravity and torsion balance data for the two limiting cases of spheres and infinite horizontal cylinders, the resolution of the vertical magnetic intensity for infinite rectangular plugs, and the direct interpretation of the infinite horizontal rectangular block. The possible usefulness of such analysis in the selection of a geophysical field method and in the choice of station spacing is discussed.

## INTRODUCTION

It is the usual practice in the interpretation of gravitational and magnetic data to work with various simple idealized shapes which may resemble the actual mass or polarization distributions only approximately. For this reason a knowledge of the properties of these simple bodies is valuable. In the present paper we establish a general method for studying the characteristics of the combined effects of two such bodies.

The problem which is considered in this paper arises in practice as follows: Suppose two equal mass anomalies are located below the surface of the ground as indicated in Fig. 1 and that the vertical gravity effect for each one is given by the solid curve; the sum of the two curves, which would be the observed gravity effect, is given by the dotted curve. If the two anomalies are far enough apart, the actual presence of two separate masses will be obvious from the appearance of the observed gravity curve. As the two masses are moved together, the gravity effects will merge and finally the two separate peaks may combine into a single peak. At this stage the gravity picture will have the deceptive appearance of a single anomaly. We shall term the separation at which the presence of two masses ceases to be visible, the resolution limit.

* Published by permission of the Gulf Research & Development Co., Pittsburgh, Pa. Paper received May 16, 1938.

† Gulf Research & Development Co., Pittsburgh, Pa.

## GENERAL METHOD OF DETERMINING THE RESOLUTION LIMIT

We shall now specialize the general idea of resolution by making the following restrictions which will cover our present purpose:

1. Resolution is considered only for profiles of effects produced by two identical bodies similarly located.

2. The effect of a single one of the bodies can be represented by $\phi(s)$, where $\phi(s)$ is a symmetric function of $s$. Here $s$ is the horizontal coordinate along the profile, with the origin $s=0$ marking the axis of symmetry of the curve.

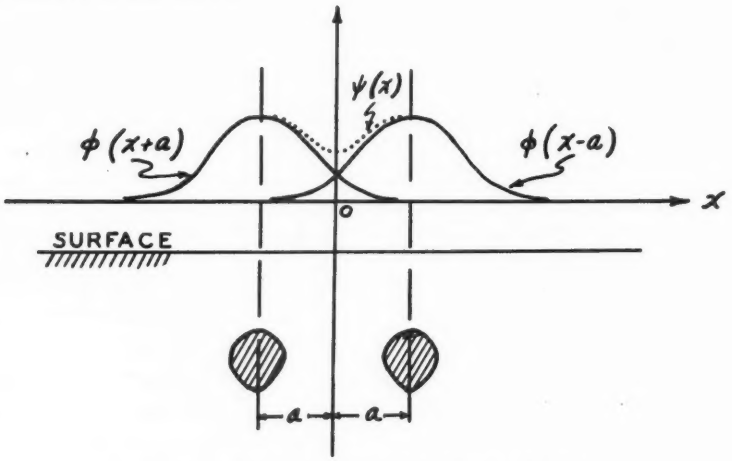



FIG. 1. Addition of effects due to identical bodies.

3. Further, the function $\phi(x)$ is an analytic function of $x$ in the neighborhood of the point $x=a$, where $a$ is the value of $x$ which determines the location of one of the bodies horizontally. Here $x$ is the horizontal coordinate along the profile measured from a fixed origin, $x=0$, which is located halfway between the two bodies.

It follows from the above assumptions that $-a$ is the value of $x$ which determines the location of the other body. Further, $\phi(x+a)$ represents the effect (at a distance $x$ along the profile) of one of the bodies, the one to the left of the origin in Fig. 1, since we take $a$ positive; while $\phi(x-a)$ represents the effect (at the same distance $x$) of the other body, the one to the right of the origin. The profile we are trying to resolve is the graph against $x$ of $\psi(x)=\phi(x+a)+\phi(x-a)$. We wish to find the values of the parameters in $\phi(x)$ at the resolution limit. We proceed as follows:

We shall denote derivatives with respect to $x$ by primes and to simplify the notation we set $\phi'(x)=f(x)$. Then, since $f(x)$ is analytic in the neighborhood of $a$, by Taylor's theorem,

$$f(x+a) = f(a) + xf'(a) + (x^2/2)f''(a) + (x^3/6)f'''(a) + \cdots$$

and $$f(x-a) = -f(a-x) = -f(a) + xf'(a) - (x^2/2)f''(a) + (x^3/6)f'''(a) - \cdots .$$

The relationship of the first two expressions in the last line follows from the symmetry of $\phi(x)$. Then the slope of the combined profile is given by

$$\psi'(x) = f(x+a) + f(x-a) = 2xf'(a) + (x^3/3)f'''(a) + \cdots . \quad (1)$$

From Fig. 2, which shows the appearance of the $\psi(x)$ profile just before

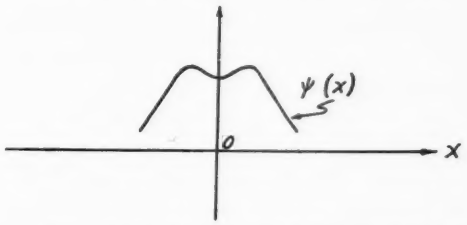



FIG. 2. Appearance of profile just before resolution limit.

the resolution limit (that is, just before the two peaks merge), it is obvious geometrically that a necessary condition for the limiting case of resolution is the occurrence of a triple tangent to the graph of $\psi(x)$ at $x=0$, that is, that equation (1) have the root $x=0$ three times. From (1) the necessary condition on the parameters for such an occurrence is $f'(a)=0$ or $\phi''(a)=0$.

The method may thus be stated as a simple working rule: Find the second derivative of the function $\phi(x)$ and set it equal to zero after substituting $a$ for $x$. The equation thus found will contain the desired condition on the parameters but it must be examined carefully to eliminate extraneous conditions since the requirement is only a necessary one. For example, Fig. 3 illustrates the situation just before another limiting case of triple tangents which would be an occurrence of one of the extraneous conditions in which we are not interested. The values of the parameters which correspond to the minimum separation of the two bodies will be the values we wish. For we may imagine the two bodies superposed (separation zero) and then

gradually separated. From equation (1) the resolution limit is the least separation at which, e.g., a dip appears in the central peak.

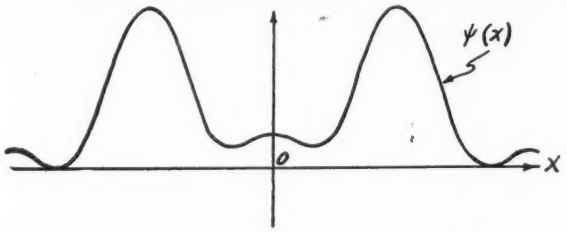



FIG. 3. Occurrence of an extraneous condition.

We mention in passing that reasoning similar to the above may be used for what might be called the anti-symmetric problem of resolution. By this we mean the case in which restrictions 1 and 2 above do not hold, but in which the curve for a single body is anti-symmetric[1] and the two curves are arranged as shown in Fig. 4. In fact, the necessary condition on the parameters is again $\phi''(a)=0$.

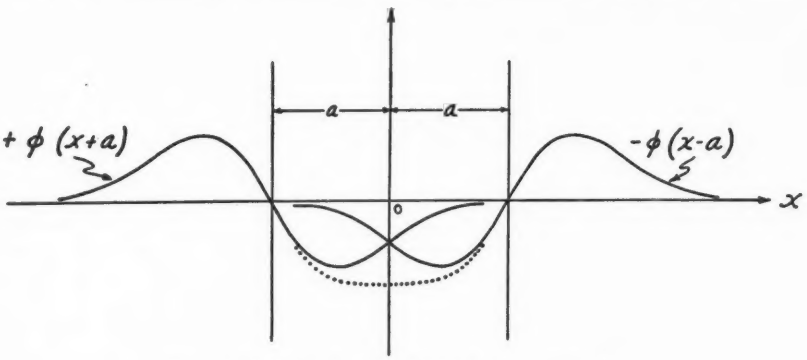



FIG. 4. Addition of effects for anti-symmetric case.

To illustrate the geophysical application of this method some examples of resolution limits will now be given. The list is not exhaustive and the reader will be able to think of additional examples.

## RESOLUTION OF TWO SPHERES BY GRAVITY AND ITS DERIVATIVES

We shall consider the resolution of a profile across the centers of two equal spheres[2] and use $x$ as the horizontal radial coordinate. We

[1] That is $\phi(s)=-\phi(-s)$, using the notation above.

[2] This restriction is made for simplicity only. The method, e.g., applies also to any profile which is parallel to the line joining the centers of the spheres.

denote by $h$ the depth to the center and by $m$ the mass of each sphere.

(a) By vertical gravity,[3] $U_z$.

The problem for this case will be worked out in detail as an example of the method. The function $\phi(x)$ is[4]

$$\frac{\gamma m h}{(x^2 + h^2)^{3/2}}.$$

The second derivative of $\phi(x)$, $\phi''(x)$, is

$$\frac{3\gamma m h(4x^2 - h^2)}{(x^2 + h^2)^{7/2}}.$$

If we substitute $a$ for $x$ in this expression and set the result equal to zero, we have

$$\frac{3\gamma m h(4a^2 - h^2)}{(a^2 + h^2)^{7/2}} = 0.$$

The solutions of this equation are obviously $a/h = \pm 1/2$, and these express the condition on the parameters at the resolution limit. This resolution condition is $2a/h = 1$. For convenience we shall denote the ratio $2a/h$ by $R$. Thus, when the distance ($2a$) between the centers of the spheres is greater than the depth ($h$) to the center of either sphere, the gravity profile shows two distinct peaks indicating the presence of two masses. When the distance between the centers is less than the depth to the center, the gravity profile shows but one peak and thus apparently indicates but one mass. The resolution limit is thus given by $R = 1$.

(b) By the vertical gradient of gravity, $U_{z^2}$.

The function $\phi(x)$ now is

$$\frac{\gamma m(2h^2 - x^2)}{(x^2 + h^2)^{5/2}}.$$

The solution of the final equation gives

$$a/h = \pm\sqrt{\frac{27 \pm \sqrt{665}}{8}}.$$

[3] We take the $z$ axis pointing vertically downward. $U$ is the gravitational potential and derivatives are denoted by subscripts. Thus

$$U_{z^n} = \frac{\partial^n U}{\partial z^n}.$$

[4] $\gamma$ is the gravitational constant.

Therefore at the resolution limit

$$R = 2a/h = \sqrt{\frac{27 - \sqrt{665}}{2}} = 0.779 \text{ approximately},$$

which corresponds to the pair of roots which has the negative sign under the radical. The two maxima are still separate for the situation given by the other pair of roots, which represents an extraneous condition.

(c) By the second vertical gradient of gravity, $U_z^3$.

The function $\phi(x)$ is

$$\frac{3\gamma m h(2h^2 - 3x^2)}{(x^2 + h^2)^{7/2}}.$$

The solution of the final equation gives

$$a/h = \pm\sqrt{\frac{41 \pm \sqrt{1393}}{36}}.$$

Therefore at the resolution limit

$$R = 2a/h = \sqrt{\frac{41 - \sqrt{1393}}{9}} = 0.639 \text{ approximately},$$

which corresponds to the pair of roots which has the negative sign under the radical. The two maxima are still separate for the situation given by the other pair of roots, as in the previous case.

As might be expected, the resolving power increases with the order of the derivative (Fig. 14).

## RESOLUTION OF TWO INFINITE HORIZONTAL CYLINDERS BY GRAVITY AND ITS DERIVATIVES

We shall consider the resolution of a profile perpendicular to the axes of two equal, parallel, infinite horizontal cylinders.[5] The $x$-axis is taken along the profile, the $z$-axis is vertically downward, while the cylinders extend indefinitely in the $\pm y$ directions. We shall denote by $h$ the depth to the center and by $m$ the mass per unit length for each cylinder.

(a) By gravity, $U_z$.

[5] This restriction is made for simplicity only. The method also applies to profiles which cross the axes of the cylinders diagonally.

The function $\phi(x)$ is

$$\frac{2\gamma mh}{x^2 + h^2}.$$

The solution of the final equation gives

$$a/h = \pm \frac{\sqrt{3}}{3}.$$

Therefore at the resolution limit

$$R = 2a/h = \frac{2\sqrt{3}}{3} = 1.155 \text{ approximately.}$$

(b) By vertical gradient of gravity,[6] $U_{z^2}$.
Here $\phi(x)$ is

$$\frac{2\gamma m(h^2 - x^2)}{(x^2 + h^2)^2}.$$

The solution of the final equation gives

$$a/h = \pm \sqrt{3 \pm \sqrt{8}}.$$

Therefore at the resolution limit

$$R = 2a/h = 2\sqrt{3 - \sqrt{8}} = 0.828 \text{ approximately,}$$

which corresponds to the pair of roots which has the negative sign under the radical. The two maxima are still separate for the situation represented by the other pair of roots, as in the case of the sphere.

(c) By the second vertical gradient of gravity, $U_{z^3}$.
Here $\phi(x)$ is

$$\frac{4\gamma mh(h^2 - 3x^2)}{(x^2 + h^2)^3}.$$

The solution of the final equation gives

$$a/h = \pm \sqrt{\frac{5 \pm \sqrt{20}}{5}}.$$

[6] The final formulae for this case as well as for case (a) have been given by H. M. Evjen, *Geophysics*, 1, 130–131, 1936.

Therefore at the resolution limit

$$R = 2a/h = 2\sqrt{\frac{5 - \sqrt{20}}{5}} = 0.650 \text{ approximately},$$

which corresponds to the pair of roots which has the negative sign under the radical. The two maxima are still separate for the situation represented by the other pair of roots, as in the previous case.

## RESOLUTION OF TWO SPHERES BY TORSION BALANCE QUANTITIES

Since the torsion balance quantities are derivatives higher by one order than gravity, one would hope to find them more sensitive indicators of resolution than gravity itself. They have therefore been investigated for spheres and infinite horizontal cylinders.

The horizontal gradient ($U_{zz}$) profile for the sphere does not satisfy the restrictions imposed in our derivations and consequently cannot be discussed by our general method. It will appear later that the gradient is of little value as a resolution criterion.

The general method can be applied to the curvature quantity ($U_\Delta$). Using the previous notation

$$\phi(x) = \frac{-3\gamma m x^2}{(x^2 + h^2)^{5/2}}.$$

The solution of the final equation gives

$$a/h = \pm\sqrt{\frac{21 \pm \sqrt{345}}{24}}.$$

Possible values at the resolution limit are given by

$$R = 2a/h = \sqrt{\frac{21 \pm \sqrt{345}}{6}} = 0.636,\ 2.568 \text{ approximately}.$$

The criterion for resolution is not very clear for this case because of the peculiar nature of the curvature profile for a sphere. However, from the curves of Fig. 5 it is possible to infer the behavior of the profile as the two spheres are moved apart. When the two spheres coincide ($R=0$), the profile is of the same appearance as that for a single sphere but with ordinates twice as great. As the spheres are moved apart, the profile keeps the same general appearance until the smaller of the two resolution limits ($R=0.636$) is reached after

which a secondary maximum appears at the center. This central maximum persists until the larger resolution limit ($R=2.568$) is reached beyond which point the maximum changes to a minimum with a maximum on each side. As the spheres are moved farther apart, the profile becomes more and more clearly that for two separate spheres.

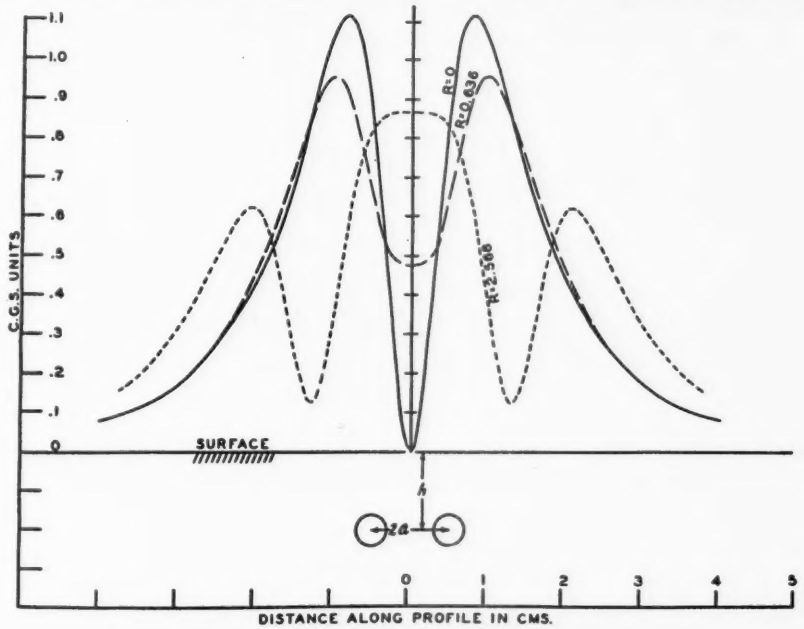



FIG. 5. Curvature profiles for two equal spheres. $h=1$, $\gamma m=-1$.

Figs. 6 and 7 show the appearance of the gravity, curvature, and gradient profiles at the resolution limit by gravity and the vertical gradient of gravity respectively. It is possible from the appearance of the curvature profiles to deduce the existence of two spheres as explained in the previous paragraph. The gradient profiles exhibit a slight flattening at the origin but this indication would be of little value with practical data. The vertical gradient profile also is given on Fig. 7.

### RESOLUTION OF TWO INFINITE HORIZONTAL CYLINDERS BY TORSION BALANCE QUANTITIES

As in the case of the sphere the horizontal gradient ($U_{zz}$) profile for the infinite horizontal cylinder cannot be discussed by our general method.

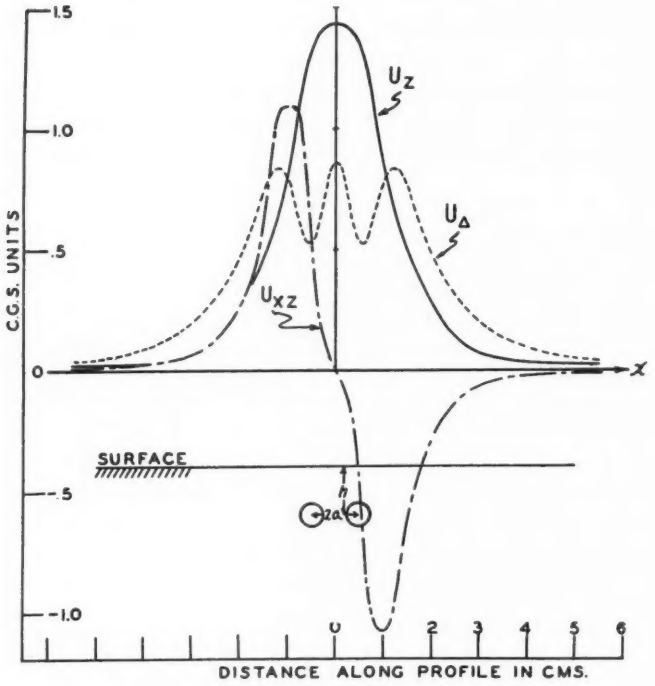



FIG. 6. Resolution of two equal spheres. Limiting case for resolution by gravity. $h=1$, $\gamma m=1$, $R=1$.

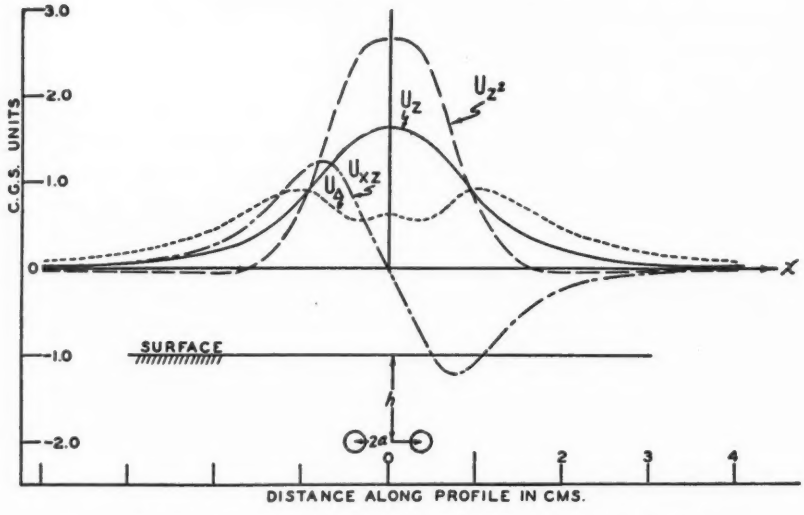



FIG. 7. Resolution of two equal spheres. Limiting case for resolution by vertical gradient of gravity. $h=1$, $\gamma m=1$, $R=0.779$.

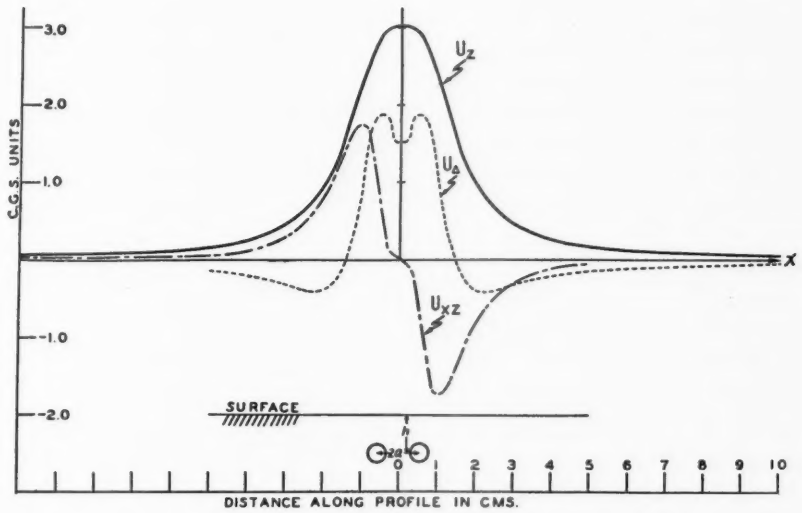



FIG. 8. Resolution of two equal infinite horizontal cylinders. Limiting case for resolution by gravity. $h=1$, $\gamma m=1$, $R=1.155$.

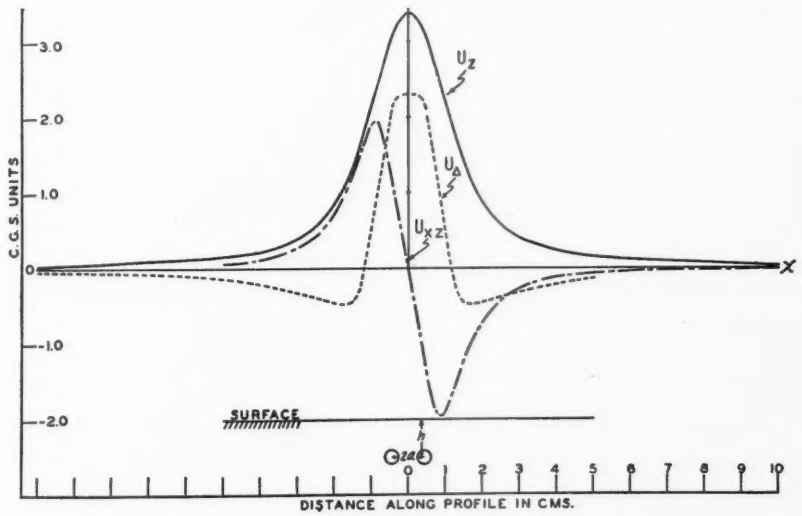



FIG. 9. Resolution of two equal infinite horizontal cylinders. Limiting case for resolution by vertical gradient of gravity. $h=1$, $\gamma m=1$, $R=0.828$.

Since for this case the $U_\Delta$ component of curvature is the same as the vertical gradient of gravity,[7] we have already solved the problem of resolution by curvatures.

Figs. 8 and 9 show the appearance of the gravity, curvature, and gradient profiles at the resolution limit by gravity and the vertical gradient of gravity respectively. The flattening of the gradient profile at the origin is an indication of little value with practical data.

## RESOLUTION OF TWO RECTANGULAR PLUGS BY VERTICAL MAGNETIC INTENSITY

We consider finally the resolution of two rectangular magnetic plugs by a profile of vertical magnetic intensity. The dimensions of the plugs are shown in Fig. 10 which is the section made by the $x$-$z$ plane perpendicular to the edges of the plugs which are supposed to be of infinite extent in the $\pm y$ directions as well as downward. The

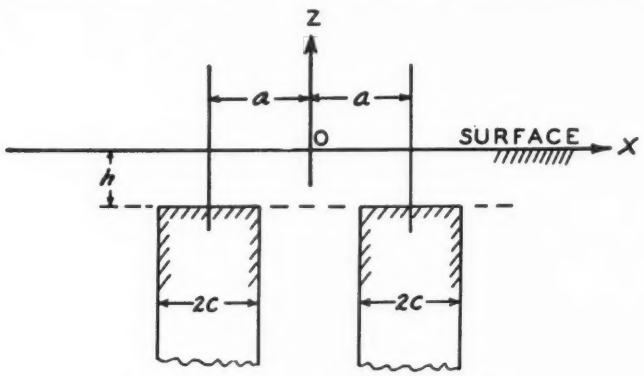



FIG. 10. Section through two magnetic plugs.

plugs are assumed to be magnetized homogeneously in the positive $z$ direction with an intensity $I$. Then for the vertical magnetic intensity

$$\phi(x) = 2I\left(\arctan\frac{x+c}{h} - \arctan\frac{x-c}{h}\right).$$

The final equation is $hc[3a^4+2(h^2-c^2)a^2-(h^2+c^2)^2]=0$. If we assume that $h$ and $c$ are not equal to zero, at the resolution limit

$$a = \pm\sqrt{\frac{c^2 - h^2 + 2\sqrt{h^4 + h^2c^2 + c^4}}{3}}.$$

[7] Laplace's equation for the two dimensional case is $U_{x^2}+U_{z^2}=0$ while $U_\Delta$ simplifies to $-U_{z^2}$. We thus have $U_{z^2}=U_\Delta$.

For the special case in which $h=10c/3$, we have the relation $R=2a/h=1.257$ approximately, at the resolution limit. Fig. 11 shows the profile of vertical magnetic intensity for this case. The cases $R=1.05$ and $R=1.5$ are also given in order to show the behavior of the profile just before and just after resolution. The profiles of vertical magnetic intensity for the separate plugs are also shown at the positions corresponding to the resolution limit.

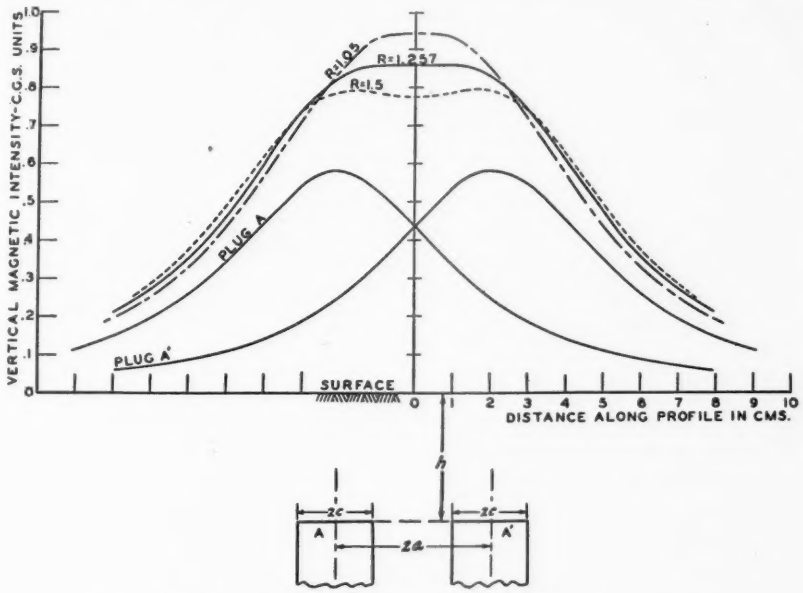



FIG. 11. Vertical magnetic intensity profiles for two equal rectangular plugs. $c=1$, $h=10/3$, $I=1/2$.

## RESOLUTION AS AN AID IN THE INTERPRETATION OF THE INFINITE HORIZONTAL RECTANGULAR BLOCK

We now give an example of the usefulness of resolution ideas in the interpretation of torsion balance data. Our previous examples have been cases in which the profile for two bodies had the deceptive appearance of the profile for a single body. The case we now consider is one in which the profile for a single body might be misinterpreted as that for two bodies.

We consider the curvature profile for an infinite horizontal rectangular block, the cross section of which is shown in Fig. 12(a). For

the present study we replace this block by two infinite faults, as shown in Fig. 12(b). For this case the function $\phi(x)$ is[8]

$$2\gamma\sigma\left(\arctan\frac{x}{h+t}-\arctan\frac{x}{h-t}\right),$$

where $\sigma$ is the density and the notation is that of Fig. 12. We have here an example of the anti-symmetric case which we mentioned

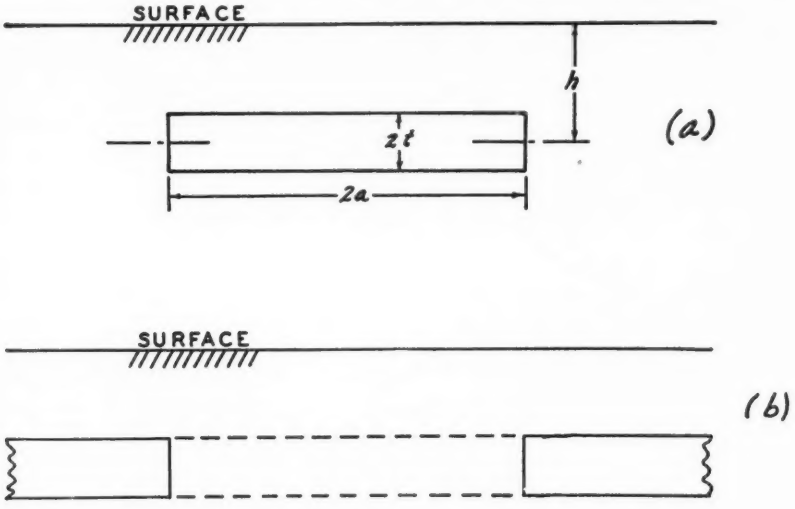



FIG. 12. Replacement of horizontal rectangular block by two infinite faults.

earlier. Applying the general method we arrive at the following equation:

$$at[a^4-2(h^2-t^2)a^2-(h^2-t^2)(3h^2+t^2)]=0.$$

The values of $a$ at the resolution limit (for $t\neq 0$) are 0 and

$$\pm\sqrt{h^2-t^2+2h\sqrt{h^2-t^2}}.$$

For the special case in which $h=1$ and $t=1/2$, we have the relation

$$R=2a/h=\sqrt{3+4\sqrt{3}}=3.151 \text{ approximately},$$

at the resolution limit. Fig. 13 shows the curvature profile for this case, and also for values of $R$ slightly less and greater.

[8] Incidentally, if we replace $\gamma\sigma$ by $I$, the same formula applies to the case of vertical magnetic intensity for vertical polarization.

We see that this resolution of the central peak provides an approximate interpretation method for cases in which we know that the structure present is of the shape of a rectangular block, for merely from the presence of one or of two peaks we can obtain from the above results an indication of the breadth of the structure. Let us consider the limiting case of a vanishingly thin block. If $t$ is small compared with $h$, $R=2\sqrt{3}=3.464$ approximately. That is, if there is but one peak, the breadth is less than about three and a half times

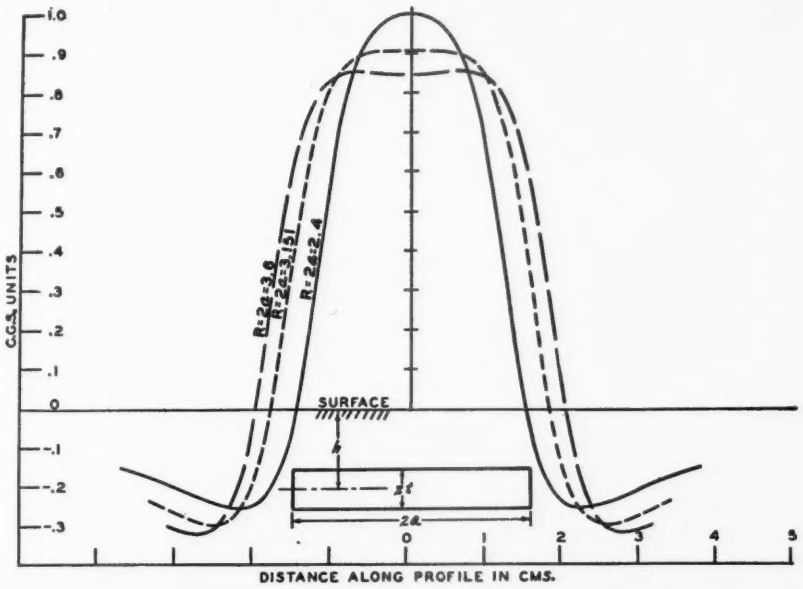


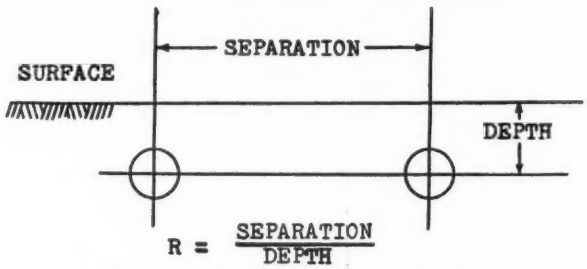

FIG. 13. Curvature profiles over infinite horizontal rectangular blocks of different breadths. $\gamma\sigma=1/2$, $h=1$, $t=1/2$.

the depth; if there are two peaks, it is greater. Since the value of $R$ changes only slightly for reasonable thicknesses (as is evident from the calculation above for $t/h=1/2$), this limiting approximation should be useful in practice for a wide variety of cases.

It should be carefully noted that a rule of this sort holds only if the underlying assumption of a single horizontal rectangular block is true. If it is desired to investigate in any particular case whether, e.g., a curve with two peaks might not be the resultant of two rectangular blocks; the method used in the earlier examples may be tried.

## RESOLUTION AS AN AID IN PLANNING A SURVEY

If the probable depth of anomalies in the area to be surveyed is known, the existence of a resolution limit will yield a value of station spacing below which there is no point in going, at least from the viewpoint of detecting individual anomalies. Again, the fact that the resolution limit varies with the quantity measured or interpreted indicates the value of the use of these resolution ideas in the choice of a method of surveying or interpreting. From these points of view Fig. 14, which lists some of the results obtained earlier in the paper,




VALUE OF $R$ AT RESOLUTION LIMIT

| | *For Cylinder* | *For Sphere* |
|---|---|---|
| By gravity, $U_z$ | 1.155 | 1 |
| By $U_{z^2}$ | .828 | .779 |
| By $U_{z^3}$ | .650 | .639 |
| By $U_\Delta$ | .828 | .636 |

FIG. 14. Some resolution limits.

will be of interest. For example, suppose gravity stations are to be observed in an area in which two-dimensional structures[9] are expected at a depth of a mile. The value of the resolution limit for a cylinder by gravity in Fig. 14 would suggest that a station spacing of the order of one mile would be adequate. On the other hand, the gravity values, no matter how accurate or closely spaced, would not resolve structures less than about a mile apart in such an area and if the finding of such closely spaced structures is the aim of the survey, a more sensitive method is required.

Actual field data will never attain the theoretical resolving power and allowance must be made for the precision of practical data in applying the above analysis.

[9] Two-dimensional structures are those in which the structural axis parallel to the surface is long in comparison with the dimensions of the cross section.

ACKNOWLEDGMENTS

We wish to express our appreciation for the assistance given by Messrs. J. Affleck, W. L. Dowler, F. W. Tinnemeyer and other members of the staff of the Gulf Research & Development Company in computation and the preparation of the figures. We are also indebted to Drs. P. D. Foote and E. A. Eckhardt for permission to publish this paper.

# THE ADJUSTMENT OF MISCLOSURES*

LAURENCE G. COWLES†

## ABSTRACT

The adjustment of misclosures in slope traverses by the method of least squares is effected by solving a system of simultaneous equations which may be written by inspection of the traverse diagram. These equations can be solved by measurements of currents in an analogous electrical resistance network. This paper is devoted mainly to the development of the electrical analogy.

## INTRODUCTION

The closing of geophysical survey traverses when the errors of measurement are appreciable necessitates the adjustment of measured data so as to absorb the observed misclosures. Any such adjustment involves, of course, the assignment of weights or relative accuracies to the observed data. The measured data are then correctly adjusted by the method of least squares in accordance with the theory of errors.

The adjustment of geophysical data by the method of least squares was discussed in detail by D. C. Barton[1] and by Irwin Roman[2] a number of years ago. One naturally expects that the method of least squares introduces complications, but as a matter of fact it is quite simple. This paper is presented to show that the least squares corrections for closing slope data traverses can be written down off-hand in terms of a system of simultaneous equations and that these equations can be solved by means of simple measurements in a suitable resistance network. The main purpose of this paper is the development of the electrical analogy, since the problem of adjustment is obvious to those familiar with the theory of least squares.

In interpreting completed seismograph surveys conducted by the slope method, one of several assumptions is that the observed slopes

* Paper read at Annual Meeting, New Orleans, La., March 16, 1938.

† The Texas Company, Houston, Texas.

The author is indebted to his colleague Dr. Alexander Wolf for assistance in preparing this paper.

[1] Control and Adjustment of Surveys with the Magnetometer or the Torsion Balance. D. C. Barton, Bulletin of the American Association of Petroleum Geologists (1929), Vol. 13, pp. 1163–86.

[2] Least Squares in Practical Geophysics. Irwin Roman, Trans. AIME (1932), Vol. 110, pp. 460–506.

actually extend over segments greater than those over which the slopes are measured. In closing traverses one must satisfy the condition that, excluding discontinuities, the line integral of the slope along any closed traverse must be equal to zero. Mathematically, if $S$ is the component of slope associated with the element of length $dl$ of the line of traverse, then,

$$\oint Sdl = 0. \tag{1}$$

The slopes which are actually measured may not satisfy equation (1) because of errors of measurement or because of faults which do not extend completely across the area surrounded by the traverse.[3] In the following discussion the possibility of faults will be excluded and the observed slopes will be adjusted by the method of least squares in such a way that equation (1) is satisfied. Parts of the survey which do not fall into a closed traverse are disregarded completely, since there is no basis for their adjustment.

## NOTATION

Consider the network of slope traverses drawn schematically in Fig. 1. The network is divided into the smallest number of loops or circuits designated by the letters $a, b, c, \ldots m$, as shown in the figure. This choice of circuits is made so that no branch of the survey belongs to more than two circuits and a standard rotation can be easily devised. Arrows are drawn in the clockwise direction to indicate the direction in which the circuits are traversed.

In a single closed traverse, of which $a$ is typical, the measured slopes, $s_{a1}, s_{a2}, \cdots$, are assumed to extend over the distances $l_1, l_2, \cdots$. The $l$'s form closed loops, which in the electrical problem will be spoken of as being made up of segments to indicate that part of a loop included between two adjacent junction points. The subscripts designate the loop and the slope number. For example, $s_{ai}$ represents slope number $i$ which is a part of loop $a$. If slope number $i$ is common to circuits $a$ and $b$, then obviously,

$$s_{ai} = -s_{bi}. \tag{2}$$

[3] Applications and Limitations of Dip Shooting. E. E. Rosaire and J. L. Adler, Bulletin of the American Association of Petroleum Geologists (1934), Vol. 18, pp. 119–132.

## GENERAL SOLUTION

The measured line integral of the slope along a closed circuit such as $a$ is

$$\int sdl = s_{a1}l_1 + s_{a2}l_2 + \cdots s_{an}l_n = A \tag{3}$$

or

$$\sum_i s_{ai}l_i = A .$$

If there were no errors in the measurements, then, obviously, $A$ would equal zero in accordance with equation (1). Because of errors $A$ is a difference in elevation, which we call the misclosure of circuit $a$. Equation (3) is called the closure equation.

After the slopes are adjusted by the method which follows, we shall have a series of corrected slopes $S_{a1}, S_{a2}, \cdots, S_{an}$, which will satisfy the relation

$$\sum_i S_{ai}l_i = 0. \tag{4}$$

We define new symbols $c_{a1}, c_{a2}, \cdots, c_{an}$ by the relations

$$S_{ai} = s_{ai} + c_{ai}. \tag{5}$$

The $c_{ai}$ are the corrections which will compensate for the misclosure $A$.

Substituting in (4) and making use of (3) we get

$$\sum_i c_{ai}l_i + A = 0.$$

This is the equation of condition for loop $A$. With $M$ circuits or loops we shall have $M$ such equations of condition:

$$\begin{aligned} &\sum_i c_{ai}l_i + A = 0 \\ &\sum_i c_{bi}l_i + B = 0 \\ &\cdots\cdots\cdots \\ &\sum_i c_{mi}l_i + M = 0. \end{aligned} \tag{6}$$

According to the theory of least squares, the most probable values of the slopes which satisfy the closure equations are obtained by selecting the corrections $c$ in such a way that a function $U$, defined by the equation

$$U \equiv \sum_N c_N^2 \tag{7}$$

is a minimum. Now $U$ is a function of all the $c$'s, but, if the total number of slopes is $N$ and the number of loops is $M$, then only $(N-M)$ of the $c$'s are independent because there are $M$ equations of condition which connect the $c$'s.

The problem of finding the maximum or minimum of several variables, not all of which are independent, is best handled by the method of undetermined multipliers, described in any textbook on the method of least squares.[4]

If the equations of condition (6) are multiplied by $2K_1$, $2K_2$, etc., respectively, and all are subtracted from $U$, then $U$ can be regarded as a function of $N$ independent $c$'s. Its minimum is obtained by partial differentiation with respect to the $c$'s. Setting the derivatives equal to zero, we thus obtain $N$ equations of the type

$$c_{xi} = K_x l_i \tag{8a}$$

or

$$c_{xi} = (K_x - K_y) l_i \tag{8b}$$

depending on whether segment $i$ is a part of one or of two circuits.

Equations (8) give the corrections in terms of the as yet undetermined multipliers $K$. To determine the values of the $K$'s we substitute the $N$ equations (8) in the $M$ equations of condition (6) and obtain a system of simultaneous algebraic equations, as follows:

$$\begin{aligned}
&\text{(Circuit } a\text{)} \quad A + K_a \sum_{a} l_i^2 - K_b \sum_{ab} l_i^2 - K_c \sum_{ac} l_i^2 \cdots - K_m \sum_{am} l_i^2 = 0 \\
&\text{(Circuit } b\text{)} \quad B - K_a \sum_{ba} l_i^2 + K_b \sum_{b} l_i^2 - K_c \sum_{bc} l_i^2 \cdots - K_m \sum_{bm} l_i^2 = 0 \\
&\text{(Circuit } c\text{)} \quad C - K_a \sum_{ca} l_i^2 - K_b \sum_{cb} l_i^2 + K_c \sum_{c} l_i^2 \cdots - K_m \sum_{cm} l_i^2 = 0 \qquad (9) \\
&\qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\text{(Circuit } m\text{)} \quad M - K_a \sum_{ma} l_i^2 - K_b \sum_{mb} l_i^2 - K_c \sum_{mc} l_i^2 \cdots + K_m \sum_{m} l_i^2 = 0
\end{aligned}$$

where $\sum_a l_i^2$ = the sum of the squares of all the $l$'s appearing in circuit $a$ and $\sum_{ab} l_i^2$ = the sum of the squares of only those $l$'s common to circuits $a$ and $b$. If there is no segment common to loops $a$ and $b$, then, obviously, $\sum_{ab} l_i^2$ = zero and this term disappears from the equations.

These equations may be set down for a particular survey by inspection of the traverse diagram. The sums of the squares of the $l$'s are obtained from the geometry of the loops, and $A$, $B$, $C$, ... $M$ are the measured misclosures given by the equations of closure (3).

[4] Practical Least Squares. O. M. Leland, McGraw-Hill, 1921.

The $M$ equations can therefore be solved simultaneously for the $M$ unknown $K$'s. The $K$'s then give the slope corrections, as shown by equations (8a) or (8b).

## SOLUTION OF THE EQUATIONS

When the number of traverses is large, the solution of the simultaneous equations (9) becomes tedious or of such difficulty as to render the application of the least squares adjustment impractical. Taking advantage of the symmetric form of the equations (9), M. H. Doolittle[4] has developed an "abridged method" for their solution, but at best the usual mathematical methods become cumbersome when there are more than ten or twenty equations.

A simpler way of solving the equations by means of an analogous electric network has been suggested by the use made of such a network by P. H. Kirkpatrick and D. G. C. Hare[5] in the solution of another problem in least squares. The electrical analogy and this method of solving the simultaneous equations by the measurement of currents in an electrical network will be illustrated by reference to a simple problem.

Represented in Fig. 1 is a four-traverse survey in which the misclosures $A$, $B$, $C$, and $D$ are to be adjusted in the loops lettered correspondingly. Application of the method of least squares as outlined above necessitates the solution of the following equations:

$$\begin{aligned}
A + K_a \sum_{a} l_i^2 - K_b \sum_{ab} l_i^2 - K_c \sum_{ac} l_i^2 - K_d \sum_{ad} l_i^2 &= 0 \\
B - K_a \sum_{ba} l_i^2 + K_b \sum_{b} l_i^2 \qquad\qquad - K_d \sum_{bd} l_i^2 &= 0 \\
C - K_a \sum_{ca} l_i^2 \qquad\qquad + K_c \sum_{c} l_i^2 - K_d \sum_{cd} l_i^2 &= 0 \\
D - K_a \sum_{da} l_i^2 - K_b \sum_{db} l_i^2 - K_c \sum_{dc} l_i^2 + K_d \sum_{d} l_i^2 &= 0.
\end{aligned} \tag{10}$$

In these the misclosures $A$, $B$, $C$, and $D$ are given and the $\sum l_i^2$ are obtained from the geometry of the traverses, so that there are four equations in the four unknown $K$'s. The slope corrections are to be secured finally by substituting the $K$'s in equations (8a) or (8b).

Now, Fig. 2 shows an electrical resistance network similar to Fig. 1 excepting that the lines of the traverse diagram have been replaced

[5] Private communication.

by resistances. With batteries having terminal voltages $A, B, C$, and $D$, as shown in the figure, the circuit equations are, of course,

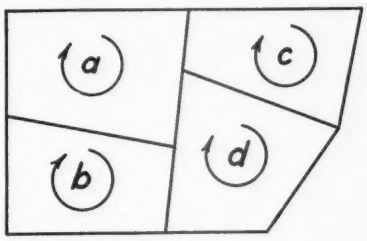



FIG. 1.

$$\begin{aligned} -A + I_a r_a - I_b r_{ab} - I_c r_{ac} - I_d r_{ad} &= 0 \\ -B - I_a r_{ba} + I_b r_b - I_d r_{bd} &= 0 \\ -C - I_a r_{ca} + I_c r_c - I_d r_{cd} &= 0 \\ -D - I_a r_{da} - I_b r_{db} - I_c r_{dc} + I_d r_d &= 0 \end{aligned} \tag{11}$$

where $r_a$ equals the sum of all resistances in loop $a$, $r_{ab}$ equals the resistance common to loops $a$ and $b$, and $I_a$, $I_b$, $I_c$, and $I_d$ are the loop currents.

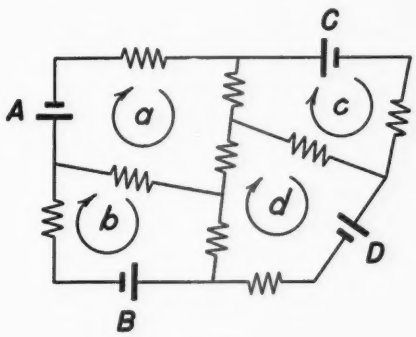



FIG. 2.

Now, if the voltages and resistances are given in the electrical problem, the currents can be found by solving the equations (11) or, conversely, one can measure the currents in the electrical network and thereby obtain the solution of the equations.

The equations (10) for the misclosure problem are similar in form to those for the electrical network, and so the measurement of cur-

rents in the electrical circuit can be used to secure the solution of the misclosure equations. Obviously, to set up this network it is only necessary to represent each segment of the seismograph survey by a resistance equal on a suitable scale to $\sum l_i^2$ for that segment, and to introduce batteries in such a way that the voltage applied in each loop is equal to the given misclosure. The currents in this circuit are then proportional to the unknown $K$'s of the closure problem.

The analogy between the misclosure problem and the electric circuit is still more striking when we note that the least squares condition—that the sum of the squares of the corrections must be a minimum—is equivalent to the well-known theorem that the currents in an electrical network flow in such a way that the total energy ($\sum I^2 r$) dissipated is a minimum.

For solving misclosure problems by this method it was found convenient to construct a number of circuit units, each consisting of a jack and variable resistances mounted on a small piece of bakelite. Once constructed, very little time is required to interconnect the units and adjust their resistances to correspond to the simultaneous equations of the misclosure problem.

In making the electrical measurements it is more practical to insert one voltage at a time, rather than all voltages at once, and to measure all the loop currents caused by the single voltage. Then, by repeating these measurements for each of the given voltages and applying the principle of superposition, the currents due to all of the voltages acting simultaneously are found by adding algebraically the currents due to the separate voltages.

The electrical method of studying misclosure problems saves considerable time and has several other advantages, especially for one who has had experience with electric circuit problems. When the number of loops is large, many currents can be neglected so that the process does not become too complicated and additional time can be saved. The corrections introduced throughout the survey by each separate misclosure can be determined by inserting one voltage at a time. Errors can be easily located, and corrections or changes in the problem do not usually require much time. Furthermore, suitable changes in the electrical network permit studies of the adjustments which may be secured by weighting data or by introducing special conditions; for example, the exact correlation of two or more points or lines of the survey.

SUMMARY

The adjustment of misclosures by the method of least squares is accomplished by solving a system of simultaneous equations which may be written down by inspection of the traverse diagram. The solution of these equations can be effected by measuring currents in an analogous electrical resistance network. The resistances of this network are determined by the geometry of the survey, and the voltages introduced in the loops are proportional to the misclosures. The substitution of the electrical analogy and some simple measurements eliminates the necessity of the laborious calculations required by the method of least squares. The latter otherwise becomes prohibitive in extensive networks.

# THREE DIMENSIONAL REFLECTION CONTROL*

S. M. ROCK†

ABSTRACT

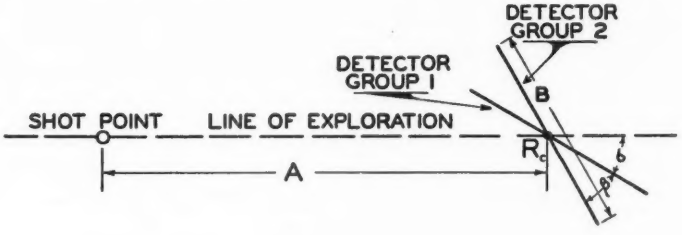
A pattern is presented in which $\Delta T$'s are obtained from intersecting lines of detectors. Assuming a) plane wave fronts at the detectors, and b) rectilinear wave propagation, formulas are presented for: (1) $\psi$, the angle of arrival of the reflected wave in the wave-travel plane: i.e., the plane through the line of exploration and perpendicular to the reflecting plane; (2) $\theta$ the angle between the wave-travel plane and a vertical plane through the line of exploration; (3) $\alpha$, the dip component in the wave-travel plane; (4) $\delta$, the total dip; and (5) $\gamma$, the angle between direction of total dip and the line of exploration.

Application of the pattern to field work is described.

Several methods are currently used to obtain three-dimensional control in seismic reflection work especially in regions of relatively high dip. However, the detector layouts employed and the methods of treating the data vary. The shooting pattern and method of treatment given below yield a "Composite Profile" from which strikes and dips may be readily calculated.

## PATTERN

Two straight-line groups of detectors intersect each other on the line of exploration a distance "A" from the shotpoint, as shown in Fig. 1. Let Group 1, which forms an angle $\sigma$ with the line of explora-




GENERALIZED SHOOTING PATTERN

FIG. 1.

* Paper read at Annual Meeting, New Orleans, La., Mar. 15, 1938, by N. C. Hunsaker.

† Rieber Laboratory, Los Angeles, California.

tion yield $\Delta T_1$. Let Group 2, which forms an angle $\beta$ with Group 1, yield $\Delta T_2$. Let $R_c$ be the intersection of the two arms of the cross. This point need not be the center of each arm, so long as corresponding waves on the two arms can be identified. Nor is it necessary that the two arms be of equal length, so long as the angle between the arriving ray and the line of detectors can be determined. However, since in field work it is customary to use two detector groups of equal length, let each arm be of length "$B$."

The following development makes the assumptions that:

1. A plane wave-front arrives at the two groups of detectors,
2. Propagation of the seismic waves in the earth is rectilinear; i.e., there is a uniform isotropic medium to any one bed.

## DEFINITIONS AND FORMULAS

Formulas may be obtained for the following angles, illustrated in Figs. 2 and 3. Consider first Fig. 2.

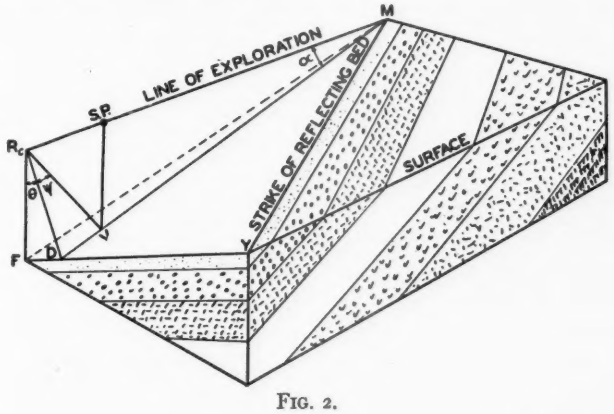



FIG. 2.

1. $\psi$, *angle of arrival.* The path of the wave received at $R_c$ lies in a plane thru the line of exploration and perpendicular to the reflecting bed. This plane ($R_cDM$) contains $R_cV$, the ray, or normal to the wave recorded at $R_c$, and is called the "wave-travel" plane. The angle of arrival, $\psi$, is measured in this plane between the normal to the line of exploration and the arriving ray.

2. $\theta$. The wave-travel plane forms an angle $\theta$ with a vertical plane through the line of exploration. $\theta$, then, is also the dip component normal to the line at the point of reflection. $\psi$ and $\theta$ for any $\sigma$ or $\beta$ are given by

$$\left.\begin{aligned} \sin\psi &= \frac{V[\Delta T_1 \sin(\sigma+\beta) - \Delta T_2 \sin\sigma]}{B \sin\beta} \\ \sin\theta &= \frac{V[\Delta T_2 \cos\sigma - \Delta T_1 \cos(\sigma+\beta)]}{B \cos\psi \sin\beta} \end{aligned}\right\}. \tag{1}$$

Except in unusual cases of extreme topographic irregularity, a $\beta$ of 90° is maintained, so that accuracy in all directions will be as nearly uniform as possible. In addition, if topography and culture permit, a $\sigma$ of either 45° or 0° is used. In these cases formulas (1) reduce to

$$\left.\begin{aligned} \sin\psi &= \frac{V(\Delta T_1 - \Delta T_2)}{\sqrt{2}\,B} \\ \sin\theta &= \frac{V(\Delta T_1 + \Delta T_2)}{\sqrt{2}\,B\cos\psi} \end{aligned}\right\} (\sigma = 45^\circ) \tag{2}$$

$$\left.\begin{aligned} \sin\psi &= \frac{V\Delta T_1}{B} \\ \sin\theta &= \frac{V\Delta T_2}{B\cos\psi} \end{aligned}\right\} (\sigma = 0). \tag{3}$$

3. $\alpha$. The wave-travel plane intersects the reflecting plane in a line which forms an angle $\alpha$ with the line of exploration. $\alpha$, then, is the component of dip in the wave-travel plane.

The reflecting bed may be located graphically by the elliptical constuction or by the familiar image method. The latter is extremely rapid when special plotting devices are employed.

$\alpha$ may also be obtained as a function of $\psi$, $A$, $T$ (total travel-time from shotpoint to the plotting surface), $V$ (the average velocity to the reflecting bed).

$$\tan\alpha = \tan\psi + \frac{A}{VT}\sec\psi. \tag{4}$$

From equation (3) and an equivalent form of (4) a universal dip chart[1] was developed. Given $\Delta T_1$, $\Delta T_2$, and $T$, $\alpha$ may be obtained

[1] Reed Lawlor, Nomogram For Dip Computations. This Journal, next paper.

readily from this chart for 1) any $A$, 2) any $B$, or 3) either any specified $V$ or any $V$ as a function of $T$.

Consider Fig. 3. $\delta$, the total dip of the reflecting bed, and $\gamma$, the direction of total dip with respect to line of exploration, are related to the angles $\alpha$ and $\theta$ by the equations:

$$\cos \delta = \cos \alpha \cos \theta \tag{5}$$

$$\tan \gamma = \frac{\sin \theta}{\tan \alpha}. \tag{6}$$

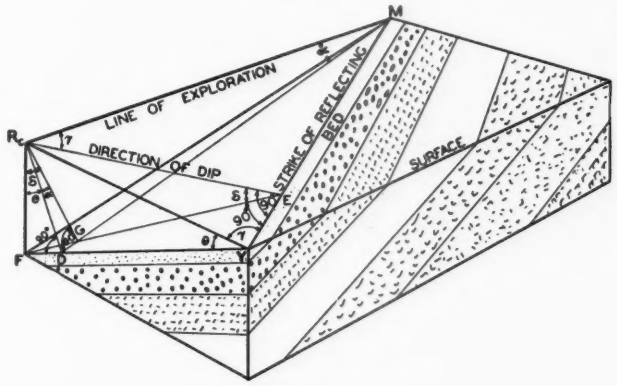



FIG. 3.

A chart designed by Curtis H. Johnson permits rapid calculation of $\delta$ and $\gamma$ from $\alpha$ and $\theta$. This chart is shown in Fig. 4. For a point with rectangular coordinates $(\alpha, \theta)$, $\delta$ is read from the family of curves which are concave toward the origin. $\gamma$ is obtained from the family of curves which are concave upward. For example, $\alpha = +10$, $\theta = -15$ yield $\delta = +18°$ and $\gamma = -56°$. This point is shown in Fig. 4.

## APPLICATIONS TO FIELD WORK

In routine field work it is often advantageous to interchange cross and shotpoint positions; i.e., to shoot reversed basements. Reflections arriving at the same time on the two setups are then compared. If they have the same $\alpha$'s and $\theta$'s, within narrow limits, they are averaged or "matched." If they disagree in either $\alpha$ or $\theta$ they are discarded. Thus, use of $\theta$ introduces an extra requirement for each "match."

The composite profile then consists of matches, on each of which is written its $\theta$. The sign convention is such that reflections from

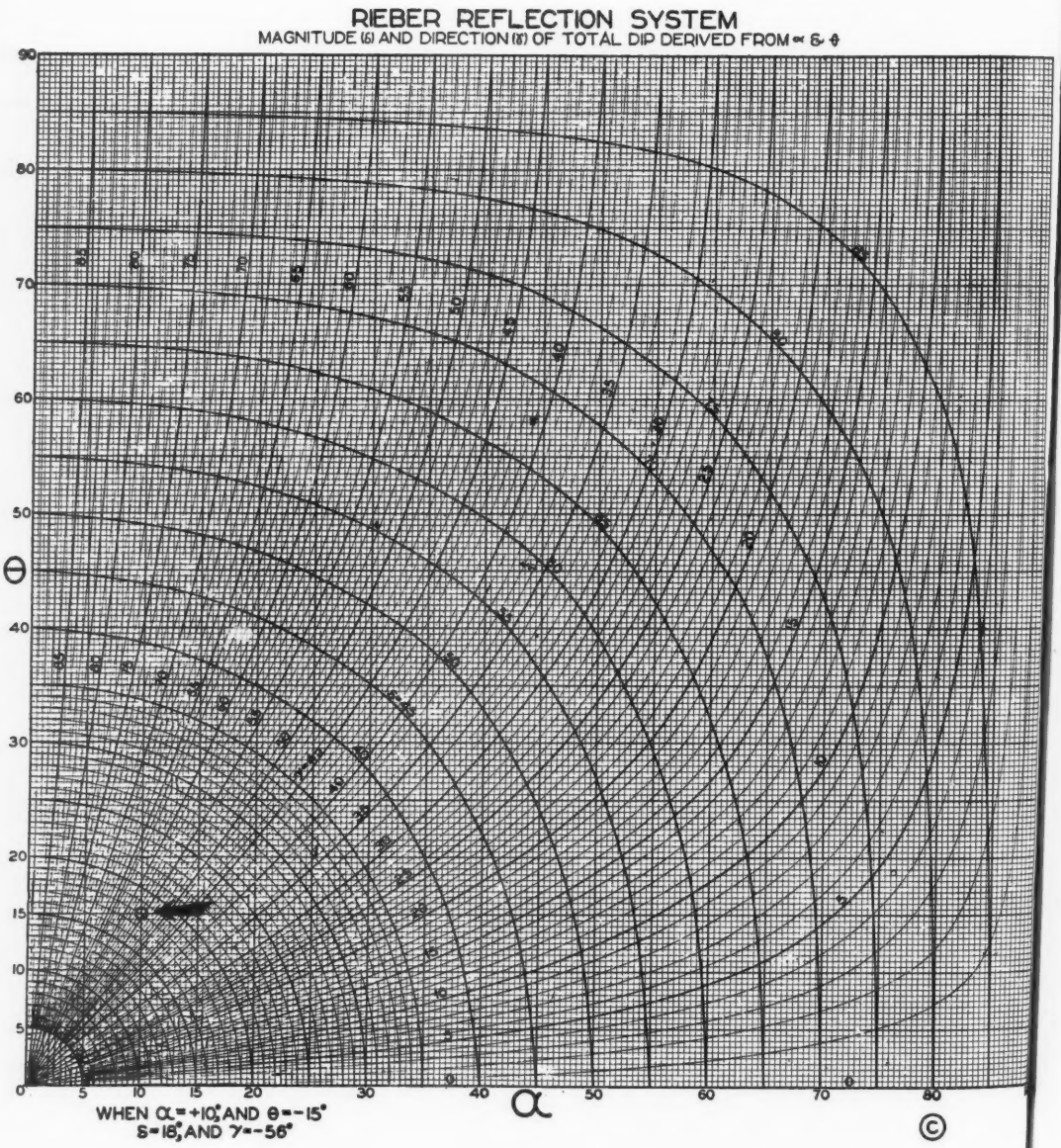

RIEBER REFLECTION SYSTEM
MAGNITUDE (δ) AND DIRECTION (θ) OF TOTAL DIP DERIVED FROM α & φ
90
80
70
60
50
θ
40
30
20
15
10
5
0
5
10
15
20
30
40
α
50
60
70
80
WHEN α=+10° AND θ=-15°
δ=18° AND γ=-56°
©

FIG. 4.

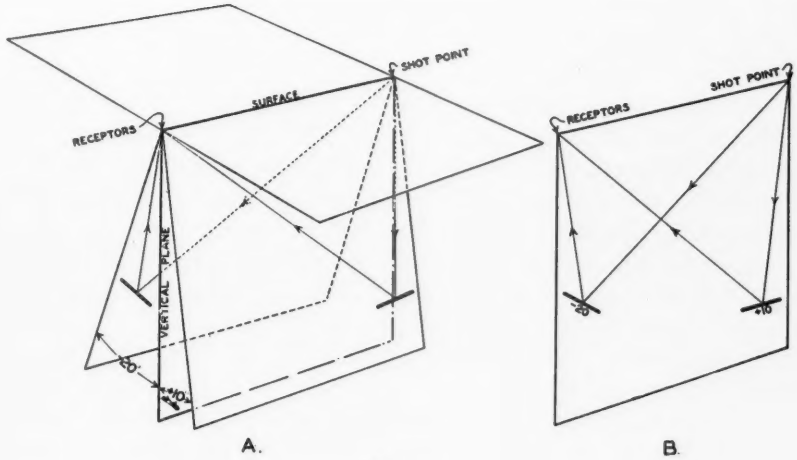
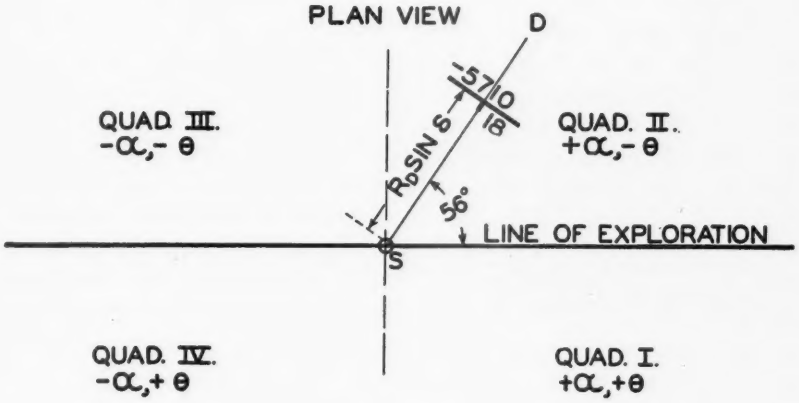



FIG. 5.




$\alpha = +10$, $\theta = -15$, $\delta = 18$, $\gamma = 56$

$R_D = 6000$, $R_D \sin \delta = 1850$, $R_D \cos \delta = 5710$

FIG. 6.

points in front of a vertical plane through the line of exploration have positive $\theta$'s, while those from behind this plane have negative $\theta$'s. Thus, on an east-west line, $+\theta$'s indicate a cross dip to the north, $-\theta$'s a cross dip to the south.

In Fig. 5A two matches, one with a $\theta$ of $-20°$, the other with a $\theta$ of $+10°$ are shown in their correct space position. Fig. 5B shows them folded into one plane, as they would appear on the composite profile. Thus, the profile itself is a three-dimensional representation of each reflecting bed.

In plotting strikes and dips from the profile, positive $\alpha$ is defined as a dip down to the west or south: i.e., down to the left side of the profile. Radial depth, $R_d$ is measured along the normal to the bed. $\gamma$, $\delta$, displacement ($R_d \sin \delta$), and depth from the plotting surface ($R_d \cos \delta$) are computed.

The quadrant in which the strike and dip symbol is to be plotted is determined by the signs of $\alpha$ and $\theta$. For example, if $\alpha = +10°$ and $\theta = -15°$, the plot is in quadrant II, using the quadrant conventions shown in Fig. 5. $\gamma$ is measured from the line of exploration in this quadrant. Thus, noting the plotting quadrant permits use of absolute values of $\alpha$, $\theta$, $\delta$, and $\gamma$, in using the chart shown in Fig. 4.

Fig. 7 shows a few groups or "trends" of matches which were obtained in California shooting. Note that groups 9, 10, 13, and 18 come from an entirely different direction in space than do 11, 29, and 24. This fact is immediately apparent from the high negative $\theta$'s of the former and the low positive $\theta$'s of the latter. This $\theta$-difference permitted grouping of waves from the same region in space. The resulting strike and dip map is shown above the profile. In the same profile, groups 25 and 27 have the same apparent $\alpha$. Yet the $\theta$'s show that the two trends are not from the same series of beds, since one group has a small cross-dip to the west while the other has a large cross-dip to the east. These two trends are also shown on the strike and dip map above the profile.

Fig. 8 shows the composite profile and strike and dip map of a portion of the shallow zone obtained from another region in California. $\theta$ angles change from negative to positive as the shooting proceeds along the line from north to south, due to an abrupt change of strike, which is considered indicative of faulting. Further evidence of the existence of this possible fault was obtained from other shooting in the vicinity.

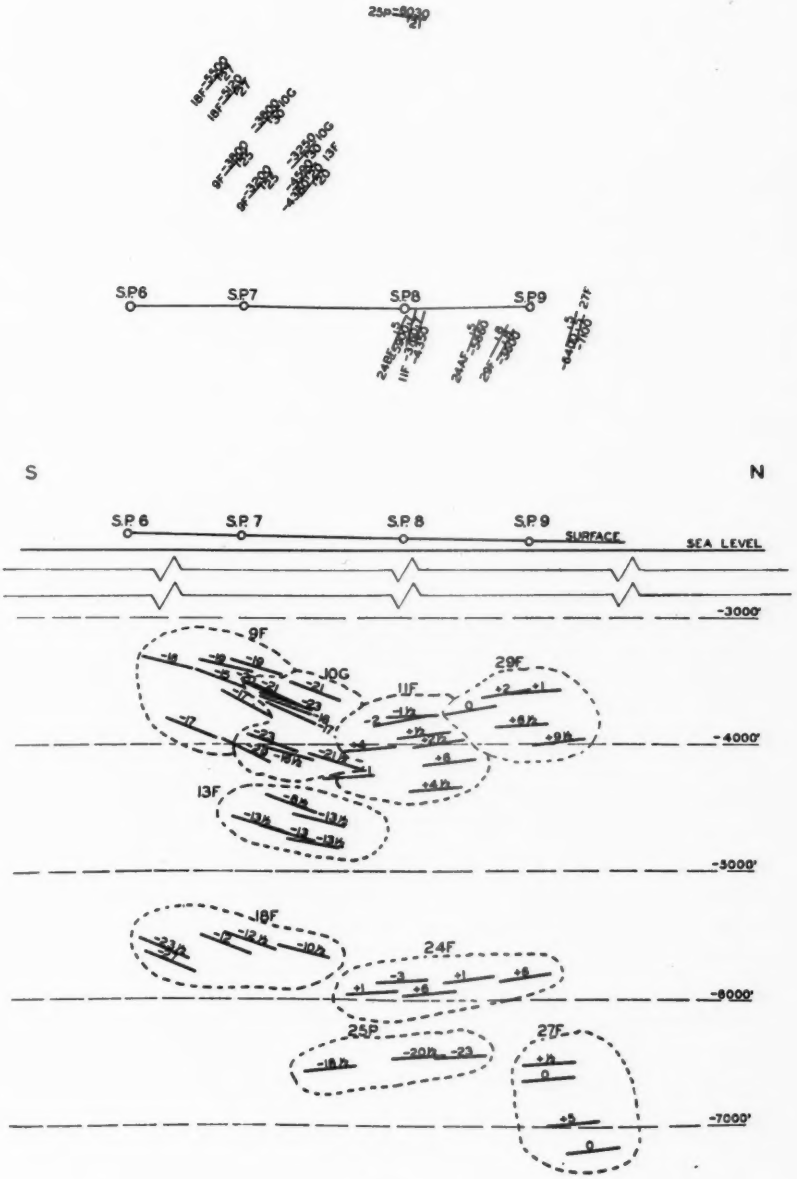

SP6
SP7
SP8
SP9
S
N
SURFACE
SEA LEVEL
-3000'
-4000'
-5000'
-8000'
-7000'
9F
10G
11F
29F
13F
18F
24F
25P
27F

FIG. 7.

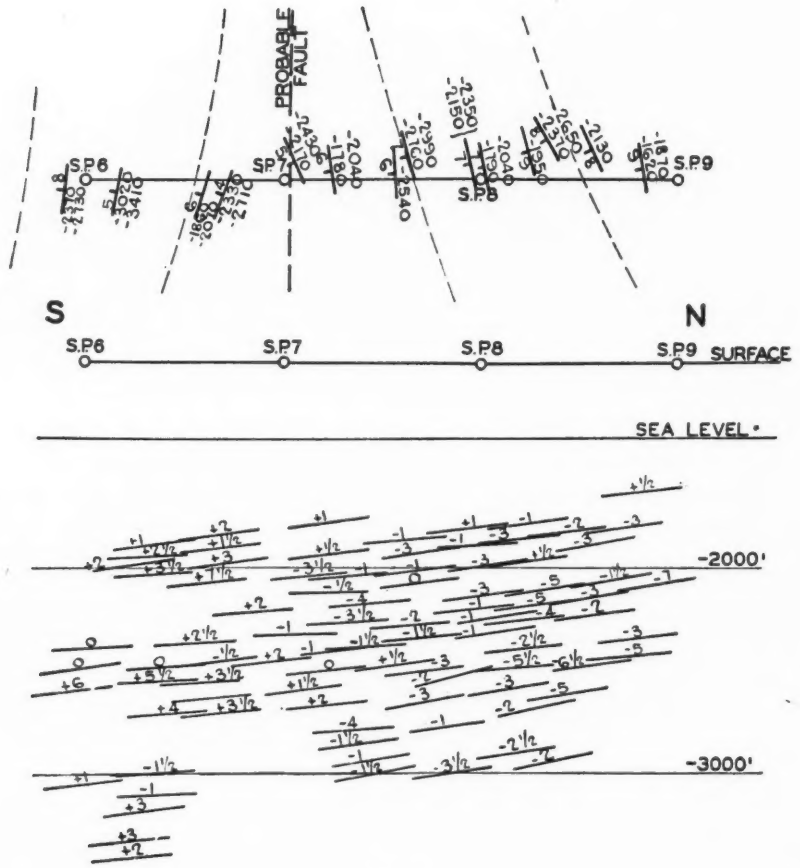



FIG. 8.

In the cases shown in Figs. 7 and 8 dips are comparatively large. However, this method of three-dimensional control has been used to advantage in regions of lower average dip. In fact, the method may be successfully applied wherever significant $\Delta T$'s are obtainable.

I wish to thank Mr. Cyrus Clark and Mr. Reed Lawlor for their criticisms and suggestions.

# NOMOGRAM FOR DIP COMPUTATIONS*

REED LAWLOR†

ABSTRACT

A universal nomogram to be used in seismographing with three-dimensional control is presented. This chart can be used for computing the dip components in and perpendicular to the wave-travel plane. By a simple change of scales it can be adapted to all shot distances, geophone spreads, and all wave velocities which are functions of wave travel time only. Principles underlying the construction of the chart are given.

The object of the present paper is to present a nomogram which can be used to calculate dip components from the field data obtained in reflection seismographing.

It has been shown[1] that the position, strike, and dip of a reflecting bed are completely determined by $\alpha$, $\theta$, $T$, and $V$, where

$\alpha$ = component of dip in the wave travel plane,
$\theta$ = angle between the wave travel plane and a vertical plane containing the line of exploration,
$T$ = travel time,
$V$ = average velocity of the seismic waves.

The universal chart presented here is applicable to all problems in which the geophones are arranged in two mutually perpendicular bisecting lines, an in-line arm with geophones in the line of exploration and a cross-line arm with geophones in a line perpendicular to the line of exploration. Frequently, shooting is conducted without the use of a cross line arm; in this case $\alpha$ is the only dip component determinable from setups along the line of exploration. In case both lines of geophones are used the chart may be used to compute $\alpha$ and $\theta$ from the $\Delta T$'s for any spread, any geophone spread, any travel time, and any velocity function provided only that the following assumptions are sufficiently accurate for interpretation of the data:

a. The average velocity is a known function of wave-travel time only, and
b. The seismic waves travel in straight lines.

* Read before the New Orleans meeting by William H. Gayman, Rieber Laboratory, Houston, Texas.

† Rieber Laboratory, Los Angeles, California.

[1] S. M. Rock, Three Dimensional Reflection Control, this Journal, preceding paper.

The equations which relate $\alpha$ and $\theta$ to the field data may be written in the forms:

$$\sin \psi = \frac{V\Delta T_1}{B} \tag{1}$$

$$\sin \theta = \frac{V\Delta T_2}{B \cos \psi} \tag{2}$$

$$\frac{\cos \alpha}{\sin (\psi - \alpha)} = \frac{VT}{A}. \tag{3}$$

Where $\psi$ = angle of arrival in the wave-travel plane,

$A$ = spread (distance from shotpoint to the intersection point of the two geophone groups)

$B$ = geophone spread (distance from first to last geophone in either line)

$\Delta T_1$ = stepout on the in-line arm.

$\Delta T_2$ = stepout on the cross-line arm.

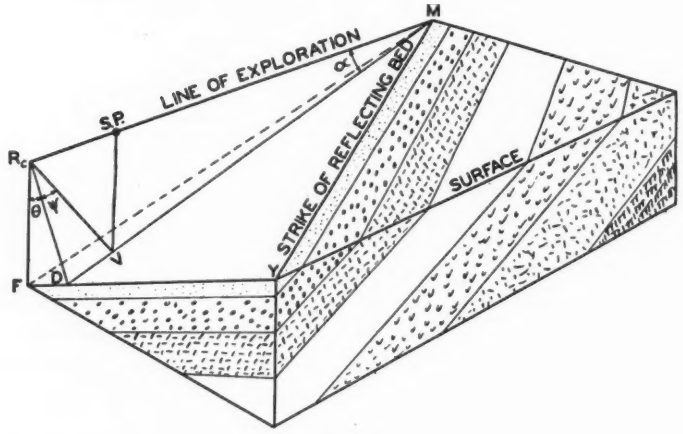



FIG. 1.

The meaning of the angles $\alpha$, $\theta$, and $\psi$ may be readily visualized by reference to Fig. 1. In this figure the line of exploration ($R_cM$) and a reflecting bed ($MYF$) are shown in relation to the surface ($MYR_c$). The shotpoint is represented by S. P. The center of the receptor group is $R_c$.

Any device which can incorporate all of the computations in-

volved in equations (1), (2), and (3) conveniently, is of use in dip shooting using both an "in-line" and a "cross-line" geophone spread or an "in-line" geophone spread alone. For lack of a better name, this computing device has been called "A Universal Dip Component Chart." Strictly speaking, the dip components as ordinarily conceived by geologists, that is, the dip components in vertical planes along and parallel to the line of exploration, are not derived directly by means of this chart. However, $\alpha$ and $\theta$, which completely determine the strike and dip, and hence the dip component in any vertical plane, are obtained directly.

Without the use of a nomogram such as that presented here, it is necessary to draw a different set of curves or compute a different table for each spread and for each velocity function encountered.

It will now be shown how each of the equations above can be mechanized and then how they are combined to form a single computing device with appropriate moving parts.

## PSI NOMOGRAM

The computation required for equation (1) may be performed readily by a rotating arm:

By rewriting 1 as

$$\sin \psi = \frac{\Delta T_1}{B} V_{\max} \cos \epsilon \tag{4}$$

where

$$\cos \epsilon = \frac{V}{V_{\max}} \tag{5}$$

and $V_{\max}$ is the highest average velocity for which computations are desired, we obtain a form suitable for mechanization.

If we let $B$ equal 500 ft. and $V_{\max}$ equal 10,000 ft. per sec. the mechanization shown in Fig. 2 is applicable.

The horizontal scale is a sin $\psi$ scale calibrated in degrees; the vertical scale is calibrated in average velocity or the travel time corresponding to the velocity. To calibrate the vertical scale it is sufficient to compute values of $\epsilon$ by means of equation (5) and write the values of velocity $V$ on the vertical scale at such a point that the angle between $OR$ and the $\psi$ axis is $\epsilon$. The travel time scale illustrated applies to the particular velocity function:

$$V = 10{,}000\,(1 - 0.4e^{-0.6T}). \tag{6}$$

The arm $OR$ is pivoted at the point $O$ and is calibrated in thousandths of a second for a geophone spread of $B = 500$ ft. To adapt this device for any other geophone spread, it is sufficient to replace the rotating arm $OR$ by another arm in which the number of $\Delta T$ units in a given length of the scale is proportional to $B$. For example, for $B = 1000$ ft. the new scale would read 100 in the distance $OR$. For convenience this arm will be referred to as the "$\Delta T$ arm."

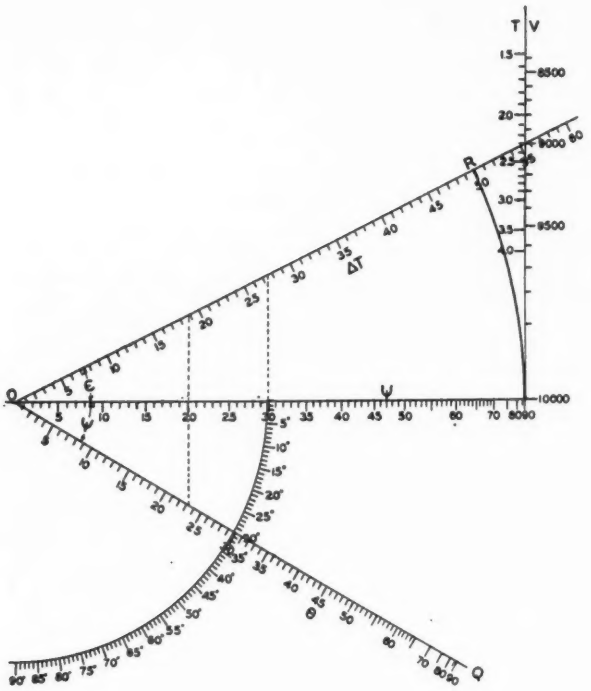



FIG. 2. $\psi$, $\theta$ Nomogram.

Computations of the angle of arrival $\psi$ by means of this device are performed as follows: First: select the $\Delta T$ scale corresponding to the particular geophone spread used ($B = 500$ ft.); second, set the $\Delta T$ arm on the appropriate velocity (9000) or corresponding travel time (2.3); and third, directly below $\Delta T$ (27.5) read $\psi$ (30°). The values in parentheses refer to the specific values illustrated in the example.

THETA NOMOGRAM

Equation (2) may be written in the form:

$$\sin \theta = \frac{\sin \psi'}{\cos \psi} \tag{7}$$

where

$$\sin \psi' = \frac{\Delta T_2}{B} V_{\max} \cos \epsilon. \tag{8}$$

To compute $\theta$, find $\psi$ as above; compute $\psi'$ in similar fashion from the given value of $\Delta T_2$; then perform the division indicated by equation (7). This division is readily performed by setting a rotating arm $OQ$ at an angle $\psi$ with the $\psi$ axis of Fig. 2 and reading $\theta$ on the $OQ$ arm directly below $\Delta T_2$ on the $\Delta T$ arm. For convenience the arm $OQ$ will be referred to as the "$\theta$ arm." Actually it is not necessary to find $\psi'$ as $\theta$ may be read on the $\theta$ arm directly below $\Delta T_2$.

Thus, to compute $\theta$ proceed as follows:

1. Find $\psi$ as above. (30°)
2. Set the $\theta$ arm at an angle $\psi$ (30°) on the circular scale.
3. Directly below $\Delta T_2$ read $\theta$ (23°) on the $\theta$ arm.

It is clear from the above discussion that Fig. 2 provides a universal chart for computing $\psi$, the angle of arrival in the wave-travel plane and $\theta$, the dihedral angle between the wave-travel plane and a vertical plane containing the line of exploration.

ALPHA NOMOGRAM

Equation (3) may be written in the form:

$$\frac{\cos \alpha}{\sin (\psi - \alpha)} = 10^Y \tag{9}$$

where

$$Y = \log V + \log T - \log A \tag{10}$$

or

$$Y = \log VT - \log A. \tag{11}$$

Equation (9) represents a family of curves of $\psi$ (or any function of $\psi$ alone) in terms of $Y$ with $\alpha$ as a parameter. This family of curves is shown in Fig. 3, with $Y = \log VT/A$ as ordinates plotted against $\sin \psi$ as abscissae. Directly below $\sin \psi$ are given the corresponding values of $\psi$. Values of the parameter $\alpha$ have been chosen at intervals of one degree, and are given in the body of the chart. To use this chart compute $\psi$ by use of the chart in Fig. 2; compute $Y$ according to equations (10) or (11) and find $\alpha$ on the chart.

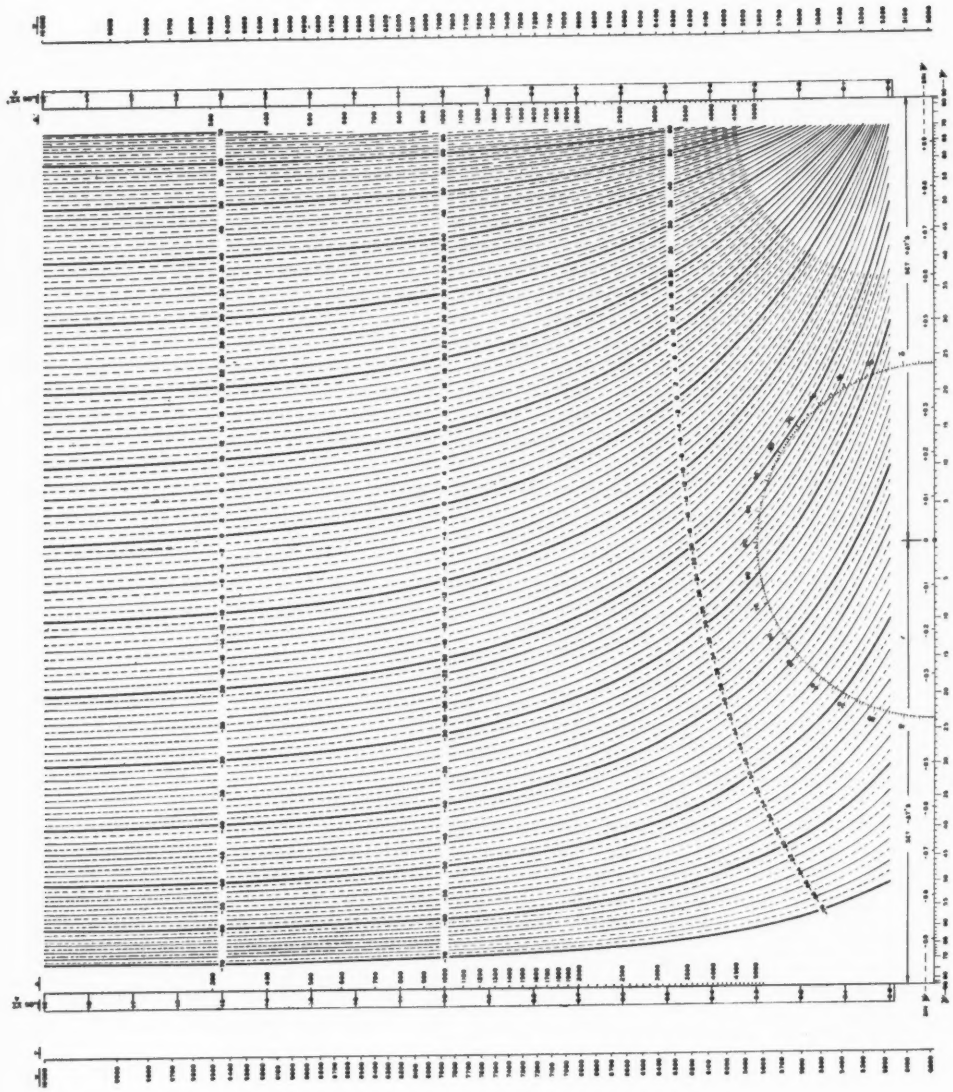


FIG. 3. Universal $\alpha$ Chart.

### Y SLIDE RULE

The computation of $Y$ may be mechanized readily as in an ordinary slide rule. This mechanization may be made for any velocity or for any velocity function.

A slide rule for computing $Y$ for any velocity in accordance with equation (10) uses the two arms (A) and (B) of Fig. 4. To compute $Y$ by means of this combination, set the V index on the (B) scale opposite $A$ on the (A) scale; then opposite $T$ on the (B) scale read $Y$ on the upper edge of (A) scale.

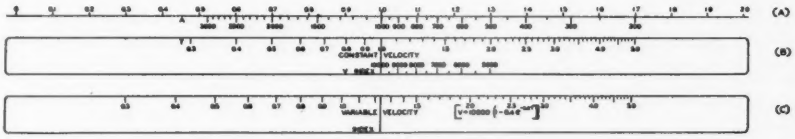



FIG. 4. $Y$ Slide Rule.

For any particular velocity function it is convenient to compute $Y$ in accordance with equation (11). Scale (C) of Fig. 4 applies to the velocity function represented by equation (6). To compute $Y$ for this velocity function, set the index on the (C) scale opposite $A$ on the (A) scale; then opposite $T$ on the (C) scale read $Y$ on the (A) scale.

To make a (C) scale for any particular velocity function, compute values of log $VT$ for that velocity function and calibrate the log $VT$ scale in terms of $T$. Set the index mark at a point corresponding to log $VT=4$.

In practice scales (B) and (C) are adjustable arms in a $T$-square as illustrated in Fig. 5. For this reason it is convenient to refer to such scales as "T-square scales."

### THE COMPOSITE NOMOGRAM

As shown in Fig. 5 the complete computing device includes the $\alpha$ chart, the $Y$ slide-rule, and the $\psi$, $\theta$ nomogram. This combination of computing devices in one nomogram is made possible by the choice of variables that has been used in the three charts. It is readily seen that the horizontal axes of the $\alpha$ chart and the $\psi$, $\theta$ chart are both calibrated in sin $\psi$ units and that the vertical scale of the $\alpha$ chart is the same as the $Y$ scale of the $Y$ slide rule.

The $\Delta T$ arm ($OR$) pivots at the point $O$ at the middle of the top of the chart. The velocity and travel time scales of Fig. 2 are at the extreme edges of the chart in Figs. 3 and 5. When $\Delta T_1$ is positive, the velocity and velocity time scales on the right edge are used; when $\Delta T_1$ is negative corresponding scales on the left are used. The $\theta$ arm

$(OQ)$ pivots at point $O'$ at the middle of the bottom of the $\alpha$ chart.

The (A) scale of the $Y$ slide rule, Fig. 4 is fixed to the chart and is designated by the term "log $VT/A$." The other arm of the $Y$ slide rule forms the long arm of the $T$-square.

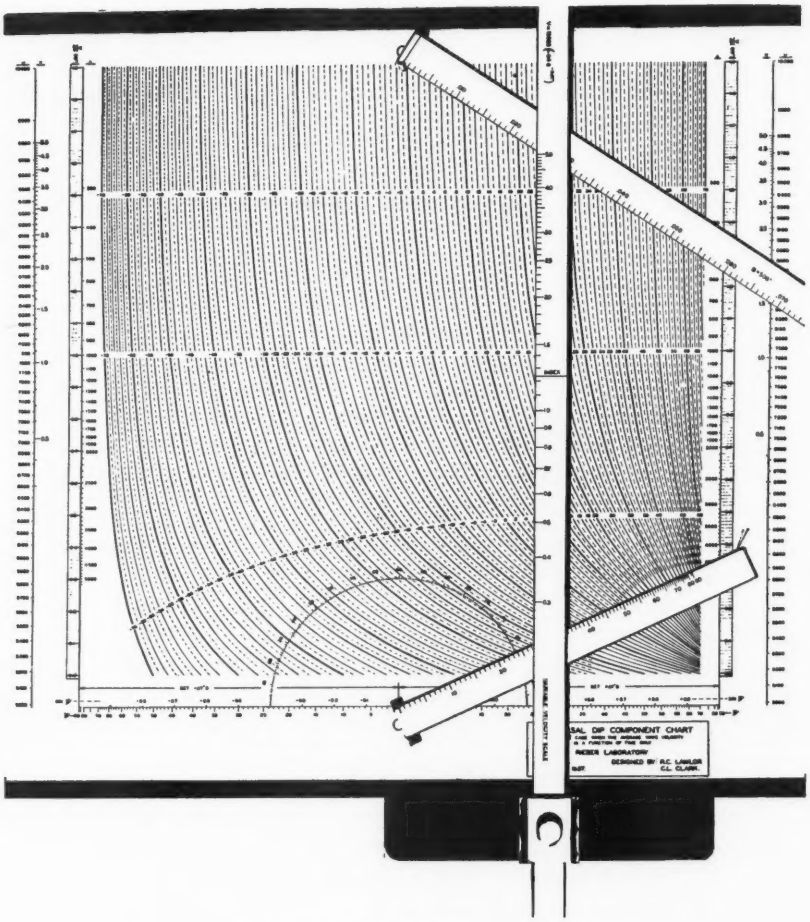


FIG. 5. Universal Dip Component Chart.

This nomogram, together with appropriate scales, can be used to compute $\alpha$ and $\theta$ for any constant velocity or for any velocity that is a function of time only. To carry out any particular computation, proceed according to the steps enumerated below. The values in parentheses refer to the particular values for which the device has been set in Fig. 5.

1. Select the time scale to be used, and mount copies of it on the side of the chart. This scale corresponds to the time scale given on the extreme edges of the $\alpha$ chart in Fig. 3.
2. Select the T-square scale corresponding to the same velocity function used in 1.
3. Select the $\Delta T$ scale appropriate to the geophone spread ($B = 500$ ft.)
4. Set the index mark of the T-square opposite the appropriate value of $A$ (1200 ft.)
5. Set the $\Delta T$ arm on the travel time (1.5 sec.) or the corresponding average velocity (8360 ft./sec.) with due regard to the sign of $\Delta T_1$.
6. Set the $T$-square on the value of $\Delta T_1$ (+0.025 sec.).
7. Opposite $T$ (1.50 sec.) on the $T$-square read $\alpha$ (19.5°)
8. Read $\psi$ (25°) on the $\psi$ scale at the bottom of the chart.
9. Set the $\theta$ arm on the appropriate value of $\psi$ (25°).
10. Set the $T$-square on the value of $\Delta T_2$ (0.025 sec. for purposes of illustration the same as $\Delta T_1$).
11. Read $\theta$ ($27\frac{1}{2}$°) on the $\theta$ arm.

While such a sequence of steps is long and tedious for the determination of $\alpha$ and $\theta$ for a single reflection, it is a very convenient device to use when a series of calculations must be made for a number of reflections involving the same velocity or velocity function, spread, and geophone spread. When such a series of calculations are required, then for each reflection only four settings (5, 6, 9, and 10) and only three readings (7, 8, and 11) are required. Only two of these three readings, namely $\alpha$ and $\theta$ need be recorded.

If, in the midst of a project, some of the geometric variables of the shooting pattern are changed, it is a simple matter to make appropriate adjustments. For example, if the spread is changed, the time scale on the $T$-square is merely shifted, if the geophone spread is changed a new $\Delta T$ scale is substituted. In changing from one project to another so that a change in velocity function is required, new velocity and time scales are calculated. Without the use of a universal dip component chart such as that presented here, each such change would require calculation of a completely new chart. It is clearly more simple to compute new scales when required than to compute new tables or new families of curves.

In conclusion I wish to thank Mr. Cyrus L. Clark for the helpful suggestions he made on the design of this nomogram, and for making the computations required for its construction.

## REVIEW

*Practical Seismology and Seismic Prospecting.* L. D. Leet. D. Appleton-Century Company, New York, 1938. x+430 pp. $6.00. (The Century Earth Science Series, Kirtley F. Mather, *Editor.*)

To most people interested in seismic prospecting, the title of the volume will perhaps be somewhat misleading. A large part of the book is devoted to earthquake lore in general; and those early chapters devoted to the geology of this subject, the reviewer found intensely interesting. On the other hand, the treatment on seismic prospecting, as the term is generally understood, is rather inadequate. One feels that the author might have done far more with the various theoretical considerations of instrumental technique, elastic theory, wave-propagation and such. The treatment, mathematical and physical, cannot be considered convincing—in fact this writer feels that it is definitely superficial in most places.

A text of this sort must naturally have a great deal of mathematical discussions. These seem to be characterized by a certain naïveté which it is difficult to describe. Perhaps it is best simply to cite a few cases: the discussions on the minimum-time path principle (pp. 110, et seq.), the successions of formulas for refraction and the derivations of these (pp. 132, et seq.), (I wonder whether anyone ever used these things in interpretations in the old refraction days!), the type set-up for hyperbolic functions (cf., in particular pp. 151 and 152), the implication that "calculus" is so *very* different (p. 193), etc., etc. On this matter, the writer feels very strongly: that it is better to omit mathematical discussions and state the desired results rather than to lead the beginner astray by sugar-coated methods which very often mislead. Certainly the author himself would agree that the seismologist who is expected to understand his science completely should be expected to have something more than a dilettante understanding of the fundamentals. Unfortunately, this naïveté permeates also the discussion of Elasticity and Elastic Waves (Part II), and other physical concepts.

Chapter 12 on the History of Seismology will prove abundantly interesting and informative.

Part V is entitled Seismic Prospecting. There are a few pertinent remarks in Chapter 13, and Chapter 14 deals with some theory of least squares and curve fitting, a discussion of fan-shooting, some statistics, and some more remarks on prospecting. The last few pages are devoted to the patent situation in the seismic field.

All in all, the reviewer is disappointed that he finds the book, insofar as the study of seismic prospecting is concerned, of no great value to the student of the subject.

M. M. SLOTNICK

# THE SOCIETY ROUND TABLE

## MEMBERSHIP APPLICATIONS APPROVED FOR PUBLICATION

The Executive Committee have approved for publication the names of the following candidates for membership in the Society. This publication does not constitute an election, but places the names before the membership at large. If any member has information bearing on the qualification of these nominees, he should send it to the Secretary within thirty days. (Names of sponsors are placed beneath the name of each nominee.)

ACTIVE

Dean Eildermann Batchelder
Merle C. Bowsky, Ralph W. Lohman, John C. Stick, Jr.

Raymond C. Clark
Geo. E. Wagoner, R. S. Dahlberg, Jr., A. B. Bryan

Forrest Elmer Curnutt
L. W. Blau, J. H. Burney, B. O. Winkler

Drexler Dana
Herbert Hoover, Jr., Henry Salvatori, C. J. Donnally

Wayne H. Denning
R. C. Coffin, Dean Walling, Harold Washburn

Arthur Edward Dietert
R. L. Kidd, G. H. Westby, A. F. Beck

Edwin Lewis Fetzer
John A. Gillin, Edward L. DeLoach, J. H. Pernell

Willis Willard Hardy
J. E. LaRue, D. P. Carlton, G. E. Bader

Alfred Julius Herzenberg
F. Goldstone, D. S. Muzzey, Jr., J. P. Woods

Ray Hauton Hillyer
R. H. Ray, J. C. Pollard, Henry Cortez

Darrell Stephan Hughes
F. Goldstone, G. W. Rulfs, D. S. Muzzey, Jr.

Alexander Ewing McKay
E. L. DeLoach, J. A. Gillin, G. C. McGhee

Allan Nordstrom
M. C. Bowsky, R. W. Lohman, J. C. Stick, Jr.

Gerrit Willem Postma
G. H. Westby, F. Goldstone, Robert Urick

Norman H. Ricker
E. E. Rosaire, L. W. Blau, W. M. Rust

Hugh C. Schaeffer
Harold Washburn, C. G. Morgan, Herbert Hoover, Jr.

Woodruff Halsey Schley
E. A. Fain, J. E. Jonsson, D. S. Renner

Paul Marie Schoorel
F. Goldstone, G. W. Rulfs, D. S. Muzzey, Jr.

Louis Perry Sexton
Henry Salvatori, J. A. Sharpe, P. F. Hawley

Daniel Silverman
Henry Salvatori, J. A. Sharpe, P. F. Hawley
Neil Roley Sparks
P. F. Hawley, J. A. Sharpe, J. D. Eisler
Hugh Miller Thralls
G. H. Westby, W. G. Green, A. J. Barthelmes
Philip Sidney Williams
A. B. Bryan, G. E. Wagoner, R. S. Dahlberg, Jr.
Lester Harold Wise
J. A. Gillin, Edward L. DeLoach, J. H. Pernell

ASSOCIATE

John Edward Backman
G. B. Lamb, E. A. Eckhardt, L. L. Nettleton
Paul Frederick Browning
B. B. Burroughs, C. H. McAllister, H. W. Rose
John Graves Cooke, Jr.
J. W. Horn, R. L. McLaren, R. A. Crain
Robert Davenport
L. Y. Faust, F. W. Borman, Maurice Ewing
Elmer Ollie Davis
J. E. Jonsson, Barney Fisher, Eugene McDermott
Charles Edward Day
F. W. Borman, L. Y. Faust, C. M. Ross
Laurence Newton Dexter
E. L. DeLoach, J. H. Pernell, Richard Williams
Edward Joseph Jasinski
G. C. McGhee, J. A. Gillin, C. G. McBurney
William George McGuire
J. C. Pollard, H. C. Cortez, T. I. Harkins
John Ross Myers
J. P. Minton, J. C. Pollard, H. C. Cortez
David Miller Weber
Richard Williams, E. L. DeLoach, G. C. McGhee

## AMENDMENTS TO THE CONSTITUTION

The following amendments to the Constitution of the Society of Exploration Geophysicists were recommended and their legality was approved by the Executive Committee. They were also approved by a favorable vote of the Business Committee. (March 16, 1938 Business Meeting)

These amendments having been approved by a favorable vote of the active membership at large, they are hereby certified as amendments to the Constitution and ordered published in the next regular issue of GEOPHYSICS in accordance with the provisions of the existing By-Laws.

| For | Against |
|---|---|
| 175 | 7 |

July 6, 1938

H. B. PEACOCK
Secretary-Treasurer

Section V-B-2

Strike out "and as the junior delegate of the Society on the Business Committee of the American Association of Petroleum Geologists."

Section V-B-4

Strike out "He shall serve as senior delegate to the Business Committee of the American Association of Petroleum Geologists."

## THE SOCIETY OF EXPLORATION GEOPHYSICISTS

Constitution and By-Laws
(As amended to July 6, 1938)

### I. NAME

This Association shall be called the "Society of Exploration Geophysicists."

### II. OBJECT

The object of this Association is to promote the science of geophysics especially as it applies to exploration.

### III. MEMBERSHIP

A—Members

1. Any geophysicist of recognized standing shall be eligible to membership, whether or not he is engaged in petroleum geophysics or in geophysical prospecting.
2. Any physicist, mathematician, geologist, or engineer of recognized standing who is investigating a geophysical problem or problems shall be eligible to membership.
3. Any geologist of recognized standing who is not a geophysicist of recognized standing, but who is acting as chief, or division chief in charge of geophysical surveying, shall be eligible to membership.

B—Associates

1. Any geophysicist, geologist, engineer, mathematician, or physicist who is a graduate of an institution of recognized scientific standing who is engaged in geophysics and who is not eligible to membership shall be eligible to associate membership.

2. Any person who is not a graduate of an institution of recognized scientific standing and who is not eligible to membership and who has shown ability in a routine phase of geophysical work and who is in a position of responsibility in geophysical surveying shall be eligible to associate membership.
3. Any representative or employee of a firm or corporation engaged in supplying equipment or supplies to the geophysical prospecting industry, provided such individual is the graduate of an institution of recognized scientific standing, or provided otherwise that such individual is qualified by adequate technical experience shall be eligible to associate membership.
4. Associate members shall be known as associates.
5. Associates shall enjoy all the privileges of membership in the Society, save that they shall not hold office, sign applications for membership, or vote; neither shall they have the privilege of advertising their affiliation with the Society in professional cards or professional reports or otherwise.
6. The Executive Committee may advance to active membership, without the formality of application for such change, those associates who have, subsequent to election fulfilled the requirements for active membership.

C—Election to Membership

1. Every candidate for admisssion as a member or associate shall submit a formal application on an application form authorized by the Executive Committee, signed by him, and endorsed by not less than three members who are in good standing, stating his training and experience and such other facts as the Executive Committee shall from time to time prescribe. Provided the Executive Committee, after due consideration, shall judge that the applicant's qualifications meet the requirements of the constitution, they shall cause to be published in an appropriate publication the applicant's name and the names of his sponsors. If, after at least thirty days have elapsed since such publication, no reason is presented why the application should not be admitted, he shall be deemed eligible to membership to to associate membership, as the case may be, and shall be notified of his election. Provided the Executive Committee may accept applicants for either class of membership, who, because of their geographical location or because of their lack of acquaintance with active members of this Society, are unable to secure qualified sponsors, but who otherwise are eligible for membership.
2. An applicant for membership, on being notified of his election in writing, shall pay full membership dues for the current year and on making such payment shall be entitled to receive the regular Society publications for that year. Unless payment of dues is made within thirty (30) days by those living within the continental United States and within ninety (90) days by those living elsewhere, after notice has been mailed of his election, the executive committee may rescind the election of the applicant. Upon payment of dues, each applicant for membership shall be furnished with a membership card for the current year and until such written notice and card are received, he shall in no way be considered a member of the Society.

D—Honorary Members

1. The Executive Committee may from time to time and by unanimous action elect

as honorary members persons who have contributed distinguished service to the cause of geophysics. Honary members shall not be required to pay dues.

E—Code of Ethics

1. Membership of any class shall be contingent upon conformance with the established principles of professional ethics.

IV

A—Resignation—Suspension—Expulsion

1. Any member or associate may resign from the Society at any time. Such resignation shall be in writing and shall be accepted by the executive committee, subject to the payment of all outstanding dues and obligations of the resigning member or associate.
2. Any member or associate who is more than one year delinquent (in arrears) in payment of dues shall be suspended from the Society. Any delinquent or suspended member or associate, at his own option, may request in writing that he be dropped from the Society and such request shall be granted by the executive committee. Any member or associate more than two years in arrears shall be dropped from the Society. The time of payment of delinquent dues for either one year or two years may be extended by a unanimous vote of the executive committee.
3. Any member or associate who resigns or is dropped under the provisions of Sections 1 and 2 of this article ceases to have any rights in the Society and ceases to incur further indebtedness to the Society.
4. Any person who has ceased to be a member or associate under Section 1 or Section 2 of this article may be re-instated by a unanimous vote of the Executive Committee subject to the payment of any outstanding dues and obligations which were incurred prior to the date when he ceased to be a member or associate of the Society.
5. Any member or associate who, after being granted a hearing by the Executive Committee, shall be found guilty of a violation of the code of ethics of this Society or shall be found guilty of a violation of the established principles of professional ethics, or shall be found guilty of having made a false or misleading statement in his application for membership in the Society, shall be asked to resign from the Society by a unanimous vote of the Executive Committee. The decision of the executive committee in all matters pertaining to the interpretation and execution of the provisions of this section shall be final.

V. Officers and their Duties

A—Officers

1. The officers of the Society shall be a President, a Vice-President, a Secretary Treasurer, and an Editor. These, together with the Past President, shall constitute the Executive Committee and managers of the Society.
2. A ticket of nomination for officers and representatives shall be prepared by a Committee on Nominations, which shall consist of the President, and the two qualified Past Presidents in order of precedence. Favorable action of this committee shall be two out of three votes. The ticket decided on by this Committee, as well as any ticket presented in writing and signed by twenty other members

in good standing, shall be submitted to the President by December 31st. At least three candidates for president shall be named on the ticket whenever the incumbent vice-president shall be ineligible for re-election. This shall be placed on a ballot and one mailed to each member in good standing on December 31st by the Secretary-Treasurer, not later than January 10th. A blank space shall be left on this ballot for writing in nominations for each place. Before ballots are mailed the President shall secure written acceptance of nomination from each candidate.

The nominations for President shall be alphabetically arranged, except the retiring Vice-President, who shall automatically occupy first place on the ticket. Each member shall express a first choice for President, and a second choice for another candidate for President. First choice votes shall count double and second choice shall count single. The candidate receiving the highest number of votes so cast and counted shall be elected President, and the candidate receiving the second highest number of votes so cast and so counted shall be elected Vice-President, except where the choice conflicts with Section Three of Article V, in which case the next highest candidate shall be elected Vice-President.

Each member shall express one choice for Secretary-Treasurer, and one choice for Editor. The candidates securing the highest number of votes so cast and counted shall be elected Secretary-Treasurer and Editor, respectively.

Such ballots to be counted as valid, must be received by the Secretary-Treasurer at his officially recognized address not less than ten days prior to the Annual Meeting. The ballots shall be mailed to the Secretary-Treasurer in envelopes which shall carry the written signature and the typewritten name of the member. These ballots will be opened and counted by the Nominating Committee immediately before the Annual Meeting. The Secretary-Treasurer, before the ballots are opened, shall approve by initiating only those ballots received from members in good standing at the time the ballots are presented to the Nominating Committee for counting.

The announcement of the election will be made as the first of the new business considered at the Annual Meeting. A majority vote of said ballots received in accordance with this action, except where otherwise designated, shall be considered as the will of the Society. In the event of a tie, the favoring secret vote of the nominating committee shall decide.

3. No one shall hold the office of President for two consecutive years and no one shall hold any other office for more than two consecutive years except the Editor, who shall not hold office for more than six consecutive years.

B—Duties of Officers

1. The President shall be the presiding officer at all the meetings of the Society, shall take cognizance of the acts of the Society and of its officers, shall appoint such committees as are required for the purposes of the Society, and shall delegate members to represent the Society. He may, at his option, serve on, and may be chairman of, any committee.
2. The Vice-President shall assume the office of president in case of a vacancy from any cause in that office and shall assume the duties of President in case of the absence or disability of the latter. He shall also act as chairman of the Program and Arrangements Committee of the Society.

3. The Secretary-Treasurer shall assume the duties of the president in case of the absence of both the President and Vice-President. He shall have charge of the financial affairs of the Society and shall annually submit reports as secretary treasurer covering the fiscal year, which he shall arrange to have published in the next regular issue of the recognized organ of the society. He shall receive all funds of the Society, and, under the direction of the Executive Committee, shall disburse all funds of the Society. He shall cause an audit to be prepared annually by a public accountant at the expense of the Society. He shall give a bond, and shall cause to be bonded all employees to whom authority may be delegated to handle Society funds. The amount of such bonds shall be set by the executive committee and the expense shall be borne by the Society. The funds of the Society shall be disbursed by check as authorized by the executive committee.

4. The Editor shall be in charge of the editorial business, shall submit an annual report of such business, shall have authority to solicit papers and material for the regular Society publication and for special publications, and may accept or reject material offered for publication. He may appoint associate, regional, and special editors.

5. In the event of absence, disability or resignation of any member of the Executive Committee, the vacant place in the Committee shall be automatically filled by the next preceding qualified Past President who shall be known as Prior Past President.

6. The officers shall assume the duties of their respective offices immediately after the Annual Meeting following their election.

## VI. Executive Committee—Meetings and Duties

A—Executive Committee

1. The Executive Committee shall consist of the President, Past President, Vice-President, Secretary-Treasurer, and Editor.

B—Meetings and Duties

2. The Executive Committee shall meet preceding the annual meeting and at the call of the president may hold meetings when and where thought advisable, to conduct the affairs of the Society. A joint meeting of the outgoing and incoming Executive Committees shall be held immediately after the close of the annual Society business meeting. Members of the Executive Committee may vote by proxy, mail or person on all matters which call for favorable or unanimous action.

3. The Executive Committee shall consider all nominations for membership and pass on the qualifications of the applicants; shall have control and management of the affairs and funds of the Society; shall determine the manner of publication; and shall designate the place of the annual meeting. They are empowered to establish a business headquarters for the Society, and to employ such persons as are needed to conduct the business of the Society. They are empowered to make investments of both general and special funds of the Society. Trust funds may be created giving to the trustees appointed for such purpose such discretion as to investments as seems desirable to the Executive Committee to accomplish any of its objects and purposes, but no such trust funds shall be created

unless they are revocable upon ninety (90) days' notice. Favorable action shall consist of a favoring three out of five votes. Committee members may vote by person, proxy or in writing.

VII

A—Meetings

1. The Society shall hold at least one stated meeting each year, which shall be the annual meeting. This meeting shall be held at a time and place designated by the Executive Committee. At this meeting the proceedings of the preceding meeting shall be read, Society business shall be transacted, scientific papers shall be read and discussed, and results announced of the mail ballot for officers for the ensuing year.

VIII

A—Amendments

1. Amendments to this constitution may be proposed by a resolution of the Executive Committee, by a Constitutional Committee appointed by the President, or in writing by any ten members of the Society. All such resolutions or proposals must be submitted at the annual meeting of the Business Committee of the Society as provided in the by-laws, and only the Business Committee shall make recommendations concerning proposed constitutional changes at the annual Society business meeting. If such recommendations by the Business Committee shall be favorably acted on at the annual Society business meeting, the Secretary-Treasurer shall arrange for a ballot of the membership by mail within thirty (30) days after said annual Society business meeting, and a majority vote of the ballots received within ninety (90) days of their mailing shall be sufficient to amend. The legality of all amendments must be determined by the Executive Committee prior to balloting.

B—Parliamentary Regulations

1. The rules contained in "Robert's Rules of Order, Revised" shall govern the Society in all cases to which they are applicable, and in which they are not inconsistent with the by-laws or the special rules of order of this Society.

BY-LAWS

I

A—Dues

1. The fiscal year of the Society shall correspond with the calendar year.
2. The annual dues of members of the Society shall be five dollars ($5.00). The annual dues of associates for not to exceed six years after election shall be four dollars ($4.00); thereafter, the annual dues of such associates shall be five dollars ($5.00). The annual dues are payable in advance on the first day of each calendar year. A bill shall be mailed to each member and associate before January first of each year, stating the amount of the annual dues and the penalty and conditions for default in payment. Members or associates who shall fail to pay their annual dues by March fifteenth shall not receive further copies of the regular Society publication, shall not be eligible to vote, nor shall they be privileged to buy Society special publications at prices made to the member ship, until such arrears are met.

II

A—Publications

1. The proceedings of the annual meeting, the constitution and by-laws, and the papers presented at such meeting shall be published at the discretion of the Executive Committee in the regular publication or in such other form as the Executive Committee may decide best meets the needs of the membership of the Society.
2. The payment of annual dues for any fiscal year entitles the member or associate to receive without further charge the regular publication of the Society for that year.
3. The Executive Committee may authorize the printing of special publications to be financed by the Society from its general, publication or special funds and offered for sale to members and associates in good standing at not less than the cost of publication and distribution.

III

A—Regional Sections, Technical Divisions, and Affiliated Societies

1. Regional Sections of the Society may be established provided the members of such sections are members of the Society and shall perfect an organization and make application to the Executive Committee. The Executive Committee shall submit the application for action first by the Business Committee and second to the Society for a letter ballot as for a constitutional amendment, and provided that the Society may revoke the charter of any regional section by similar action.
2. The Executive Committee may arrange for the affiliation with the Society of duly organized groups or societies, which by object, aims, constitution, by-laws, or practice are developing the study of geophysics or petroleum technology. In like manner, the Executive Committee may arrange conditions for dissolution of such affiliations. Affiliation with the Society need not prevent affiliation with other scientific societies. Members of affiliated societies who are not members of the Society, shall not have the privilege of advertising their affiliation with the Society on professional cards or otherwise, but shall be entitled to receive the regular publication of the Society upon payment of a fee equivalent to that charged an associate member.

IV

A—District Representatives

1. The Executive Committee may cause to be elected district representatives from districts which it shall define by a local geographic grouping of the membership. Such districts shall be redesignated and redefined by the Executive Committee as often as seems advisable. Each district shall be entitled to one representative for each seventy-five members, but this shall not deprive any designated district of at least one representative. The representatives so apportioned shall be chosen from the membership of the district by a written ballot arranged by the Executive Committee. They shall hold office for two years, their term of office expiring at the close of the annual meeting. In so far as possible, these two-year terms shall be overlapping.

V

A—Business Committee

1. The Executive Committee may institute a Business Committee to act as a council and advisory board to the Executive Committee and the Society. This Committee shall consist of the Executive Committee, the Prior Past President, and the district representatives. Favorable action by the Business Committee shall consist of a favorable two-thirds vote. The Secretary-Treasurer shall act as Secretary of the business meeting. Members of this Committee may vote in person, by proxy, or in writing. The Business Committee shall meet the day before the annual meeting at which time proposed changes in the constitution or by-laws may be considered, all old and new business shall be discussed, and recommendations may be voted for presentation at the annual meeting.

VI

A—Amendments

1. These by-laws may be amended by favorable vote of the Business Committee, providing that such changes have been recommended and legality approved by the Executive Committee. Such amendments shall be published in the next regular number of the recognized organ of the Society.